Lecture 6:

Oligopoly

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Games and Decisions
Market Structures

- the list of basic market structure types (seller-side types only):

<table>
<thead>
<tr>
<th></th>
<th>Number of sellers</th>
<th>Seller entry barriers</th>
<th>Deadweight loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect competition</td>
<td>Many</td>
<td>No</td>
<td>None</td>
</tr>
<tr>
<td>Monopolistic competition</td>
<td>Many</td>
<td>No</td>
<td>None</td>
</tr>
<tr>
<td>Oligopoly</td>
<td>Few</td>
<td>Yes</td>
<td>Medium</td>
</tr>
<tr>
<td>Monopoly</td>
<td>One</td>
<td>Yes</td>
<td>High</td>
</tr>
</tbody>
</table>
Collusive vs. Non-Collusive Oligopolies

- note: oligopoly differs from monopoly (allocation-wise) only if there’s no *collusion*
  - collusion: a largely illegal form of cooperation amongst the sellers that includes price fixing, market division, total industry output control, profit division, etc.
  - controlled by competition/anti-trust laws
  - well-known collusion cases: OPEC, telecommunication, drugs, sports, chip dumping (poker)

- game-theoretical models:
  - cooperative setting (*collusive oligopoly*) → coalition theory
    - games in the *characteristic-function form*
  - non-cooperative setting (*competitive, non-collusive oligopoly*) → normal form game analysis
    - NE’s etc.; however, matrices can’t typically be used for payoffs
Oligopoly – Model Specification

- to make the analysis simple, we’ll make several assumptions:
  1. *single-product model*: oligopolists produce a single type of *homogenous* product
  2. *one strategic variable*: firms decide about prices or output levels
  3. *static model*: single-period analysis only
     - in dynamic models, there are more diverse strategic options: elimination of competitors even with contemporary losses etc.
  4. *single objective*: all firms maximize their individual profit

**Three basic non-cooperative oligopoly models:**
- *Bertrand* oligopoly – firms simultaneously choose prices
- *Cournot* oligopoly – firms simultaneously choose quantities
- *Stackelberg* oligopoly – firms choose quantities sequentially
  - note: sequential-move games are typically not modelled as normal-form games. Instead, we use the extensive-form approach (not this lecture).
Bertrand Duopoly

- Bertrand duopoly (2 oligopolists only) – model notation:
  - market demand function: \( q = D(p) \)
  - prices charged by the players: \( p_1, p_2 \)
  - resulting quantities: \( q_1, q_2 \)
  - unit costs: \( c_1, c_2 \) (for simplicity: \( AC = MC = c \))

- homogenous product → lower price attracts all the consumers
  - \( p_1 < p_2 \) → \( q_1 = D(p_1), \quad q_2 = 0 \)
  - \( p_1 > p_2 \) → \( q_1 = 0, \quad q_2 = D(p_2) \)
  - \( p_1 = p_2 \) → equal market share, \( q_1 = q_2 = \frac{1}{2}D(p_1) = \frac{1}{2}D(p_2) \)

- as long as the prices are higher than \( c_1 \) and \( c_2 \), both oligopolists tend to push prices down (below the other player’s price)
  - imagine the prices are equal and above \( c_1 \); by lowering the price just slightly, player 1 can gain the whole market (if \( p_2 \) stays the same)
  - best response of player 1 to \( p_2 \) is to choose \( p_1 = p_2 - \varepsilon \) (“just below” \( p_2 \))
    (until the prices reach \( c_1 \) → player 1 suffers a loss below)
NE depends on the MC of the players:

- \( c_1 = c_2 \rightarrow p_1^* = p_2^* = c_1 = c_2 \) → zero economic profit for both
- \( c_1 < c_2 \rightarrow p_1^* = c_2 - \varepsilon \) → player 1 wins all, positive profit
- \( c_1 > c_2 \rightarrow p_2^* = c_1 - \varepsilon \) → player 2 wins all, positive profit

- more precisely: graphical best-response analysis – reaction curves:
→ price competition leads to fairly efficient allocation

**Critique of the Bertrand model** (or, when Bertrand model fails to work)

- capacity constraints of production
  - e.g., consider the $c_1 < c_2$ situation: if player 1 can’t supply enough for the whole market, player 2 can still charge $p_2$ above $c_2$ and attract some customers (and achieve a positive profit)
  - if $c_1 = c_2$ and neither player can supply to all customers, either player can raise the output price above $c$

- lack of product homogeneity (homogeneity disputable in most cases)

- transaction/transportation costs:
  - may differ for the specific customer–firm interactions
    - e.g., shops at both ends of a street: people tend to pick the closer one
    - if transportation costs are accounted for, the consumer expenditures vary even if prices are equal
Cournot Oligopoly – Formal Treatment

- model type – normal-form game with the following elements:
  - list of firms: 1,2,...,N
  - strategy spaces: \( X_1, X_2,...,X_N \)
    - potential quantities: typically intervals like [0,1000] → infinite alternatives!
    - the output level produced by i\(^{th}\) player (the strategy adopted) is denoted \( x_i \)
    - a strategy profile is an \( N \)-tuple: \((x_1,x_2,...,x_N)\) (where \( x_i \in X_i \))
  - cost functions: \( c_1(x_1), c_2(x_2),... , c_N(x_N) \)
    - total cost as the function of output level
  - price function (or inverse demand function): \( p = f(x_1 + x_2 + ... + x_N) \)
    - i.e., market price is the function of total industry output

- profit of i\(^{th}\) firm: \( \pi_i(x_1,...,x_N) = TR_i - TC_i = x_i \times f(x_1 + ... + x_N) - c_i(x_i) \)
Nash Equilibrium in Cournot Oligopoly

- **Mathematical definition:**
  
  A strategy profile \((x_1^*, x_2^*, \ldots, x_N^*)\) is a NE if for all \(i = 1, \ldots, N\)
  
  \[ \pi_i(x_1^*, x_2^*, \ldots, x_i, \ldots, x_N^*) \leq \pi_i(x_1^*, x_2^*, \ldots, x_i^*, \ldots, x_N^*) \]

  holds for all \(x_i \in X_i\).

- Finding the NE: best-response approach (again)
  - NE: the strategies have to be the best responses to one another
  - Best-response functions:
    - For player 1: \(r_1(x_2, \ldots, x_N)\) is the best-response \(x_1\) chosen by player 1, given that the other player's quantities are \(x_2, \ldots, x_N\)
    - Mathematically: \[ r_1(x_2, \ldots x_N) = \arg\max_{x_1 \in X_1} \pi_i(x_1, x_2, \ldots, x_N) \]
    - NE: \(x_i^* = r_i(x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_N^*)\) for \(i = 1, \ldots, N\)
Example 1: Cournot Duopoly

- price function: \( p = f(x_1 + x_2) = 100 - (x_1 + x_2) \)

- other characteristics: \( X_1 = [0, +\infty) \) \( c_1(x_1) = 150 + 12x_1 \)
  \( X_2 = [0, +\infty) \) \( c_2(x_2) = x_2^2 \)

- profit functions: \( \pi_1(x_1, x_2) = x_1 \times f(x_1 + x_2) - c_1(x_1) = \)
  \( = x_1 \times [100 - (x_1 + x_2)] - (150 + 12x_1) = \)
  \( = 100x_1 - x_1^2 - x_1x_2 - 150 - 12x_1 = \)
  \( = 88x_1 - x_1^2 - x_1x_2 - 150 \)
  \( \pi_2(x_1, x_2) = \ldots = 100x_2 - 2x_2^2 - x_1x_2 \)

- best response of player 1 to \( x_2 \): profit-maximizing (\( \pi_1 \)-maximizing) value of \( x_1 \) for the given \( x_2 \)
Example 1: Cournot Duopoly

- profit of player 1 for three different levels of $x_2$:

  ![Graph showing profit of player 1 for three different levels of $x_2$.]

  - best response to $x_2 = 10$ is $x_1 \approx 40$,
  - best response to $x_2 = 25$ is $x_1 \approx 33$,
  - best response to $x_2 = 50$ is $x_1 \approx 20$,

- best response for an arbitrary level of $x_2$: as the function $\pi_1(x_1, x_2)$ is strictly concave in $x_1$ for any $x_2$, we can use the first-order condition for a local extreme

\[
\frac{\partial \pi_1(x_1, x_2)}{\partial x_1} = 88 - 2x_1 - x_2 = 0
\]
Note: first order conditions are generally not sufficient for a maximum, only necessary conditions for extreme (but: concave function $\rightarrow$ global maximum)

$$\frac{\partial f(x)}{\partial x} = 0$$
we can also write the result in terms of the reaction function $r_1$:

$$\frac{\partial \pi_1(x_1, x_2)}{\partial x_1} = 88 - 2x_1 - x_2 = 0 \implies x_1 = r_1(x_2) = 44 - \frac{x_2}{2}$$

similarly, for player 2, we obtain:

$$\frac{\partial \pi_2(x_1, x_2)}{\partial x_2} = 100 - x_1 - 4x_2 = 0 \implies x_2 = r_2(x_1) = 25 - \frac{x_1}{4}$$

altogether, we have 2 linear equations; for NE strategies, both have to hold at the same time → in order to find the NE, we just need to solve

$$88 - 2x_1^* - x_2^* = 0 \quad \text{or} \quad x_1^* = r_1(x_2^*) \quad \Rightarrow \quad x_1^* = 36$$

$$100 - x_1^* - 4x_2^* = 0 \quad \text{or} \quad x_2^* = r_2(x_1^*) \quad \Rightarrow \quad x_2^* = 16$$

Question:
What are the equilibrium profits and price?
Collusive Oligopoly

- **model framework**: as in case of Cournot oligopoly, only that players can form coalitions
- **coalition**: a group of firms that coordinate output levels and redistribute profits
- **grand coalition**: the coalition of all oligopolists, \( Q = \{1, 2, \ldots, N\} \)
  - other coalitions are denoted by \( K, L, \ldots \)
  - a “single-firm coalition” is still called a coalition, e.g. \( K = \{2\} \), and so is the “empty coalition” \( \emptyset \)

**Question:**
How many different coalitions can be formed with \( N \) firms?

- **characteristic function** (of the oligopoly): a function \( v(K) \) that assigns to any coalition \( K \) the maximum attainable total profit of \( K \)
  - **payoff function**: single player, individual payoff, for a given strategy profile
  - **characteristic function**: coalition, sum of members’ profits, max. attainable
characteristic function for grand coalition:

\[ u(Q) = \max_{(x_1, \ldots, x_N)} \sum_{i=1}^{N} \pi_i(x_1, \ldots, x_N) \]

characteristic function for other coalitions: profit of coalition members depends on the quantity chosen by non-members

→ what will the other players do? (Generally, difficult to say.)

1. **minimax characteristic function**: assume non-members supply as much as they can (up to their capacity constraints)

2. **equilibrium characteristic function**: assume the other players choose the NE quantities

characteristic function for an arbitrary coalition:

\[ u(K) = \max_{(x_i)_{i\in K}} \sum_{i\in K} \pi_i(x_1, \ldots, x_N) \]
Example 2: Collusive Duopoly

- consider the same duopoly as in example 1, only with capacity constraints:
  - price function: \( p = f(x_1 + x_2) = 100 - (x_1 + x_2) \)
  - other characteristics: \( X_1 = [0, 40] \) \( c_1(x_1) = 150 + 12x_1 \)
    \( X_2 = [0, 20] \) \( c_2(x_2) = x_2^2 \)
  - profit functions: \( \pi_1(x_1, x_2) = 88x_1 - x_1^2 - x_1x_2 - 150 \)
    \( \pi_2(x_1, x_2) = 100x_2 - 2x_2^2 - x_1x_2 \)

- first, we’ll find the equilibrium characteristic function:
  - we already know the NE values: \( x_1^* = 36 \) \( \pi_1^* = 1146 \)
    \( x_2^* = 16 \) \( \pi_2^* = 512 \)
  - immediately, we have: \( v(1) = \max_{x_1 \in X_1} \pi_1(x_1, 16) = \pi_1(36, 16) = 1146 \)
    \( v(2) = \max_{x_2 \in X_2} \pi_2(36, x_2) = \pi_2(36, 16) = 512 \)
Example 2: Collusive Duopoly

- for grand coalition \( Q = \{1,2\} \), we obtain:

\[
v(1,2) = \max_{(x_1, x_2)} \pi_1(x_1, x_2) + \pi_2(x_1, x_2) =
\]

\[
= \max_{(x_1, x_2)} 88x_1 + 100x_2 - x_1^2 - 2x_2^2 - 2x_1x_2 - 150
\]

- function of two variables now, but still concave (see next slide)
→ first-order conditions (both partial derivatives equal zero)

\[
\frac{\partial \pi_{1,2}(x_1, x_2)}{\partial x_1} = 88 - 2x_1 - 2x_2 = 0
\]

\[
\frac{\partial \pi_{1,2}(x_1, x_2)}{\partial x_2} = 100 - 2x_1 - 4x_2 = 0
\]

\[
\Rightarrow \begin{cases} 
  x_1^{opt} = 38 \\
  x_2^{opt} = 6 
\end{cases} \quad \pi_{1,2}^{opt} = 1822
\]

- \( v(Q) = v(1,2) = 1822 \)
Example 2: Collusive Duopoly (cont’d)
first-order conditions: necessary conditions for local extremes *(not sufficient, not for maxima only!)*
Example 2: Collusive Duopoly

- The complete *equilibrium characteristic function* is as follows:
  \[
  v(\emptyset) = 0 \\
  v(1) = 1146 \\
  v(2) = 512 \\
  v(1,2) = 1822
  \]

- *Minimax characteristic function*:
  - \(v(\emptyset)\) and \(v(1,2)\) are the same as in the equilibrium char. function
  - For \(v(1)\) and \(v(2)\), we calculate the players’ profits under the condition that the other player produces up to his/her capacity constraint:

  \[
  v(1) = \max_{x_1 \in X_1} \pi_1(x_1,20) = \max_{x_1 \in X_1} 68x_1 - x_1^2 - 150 = \pi_1(34,20) = 1006 \\
  v(2) = \max_{x_2 \in X_2} \pi_2(40,x_2) = \max_{x_2 \in X_2} 60x_2 - 2x_2^2 = \pi_2(40,15) = 450
  \]
Example 2: Collusive Duopoly

- a comparison of the two characteristic functions:

<table>
<thead>
<tr>
<th></th>
<th>equilibrium</th>
<th>minimax</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v(\emptyset)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$v(1)$</td>
<td>1146</td>
<td>1006</td>
</tr>
<tr>
<td>$v(2)$</td>
<td>512</td>
<td>450</td>
</tr>
<tr>
<td>$v(1,2)$</td>
<td>1822</td>
<td>1822</td>
</tr>
<tr>
<td>$v(1,2) - v(1) - v(2)$</td>
<td>164</td>
<td>366</td>
</tr>
</tbody>
</table>

- core of the oligopoly: a division of payoffs $a_1,a_2$ such that

  $a_1 + a_2 = 1822,$
  
  $a_1 \geq 1146,$ or $a_1 \geq 1006,$
  
  $a_2 \geq 512,$ or $a_2 \geq 450.$
Lecture 6: Oligopoly

Jan Zouhar Games and Decisions