Lecture 6:
Oligopoly

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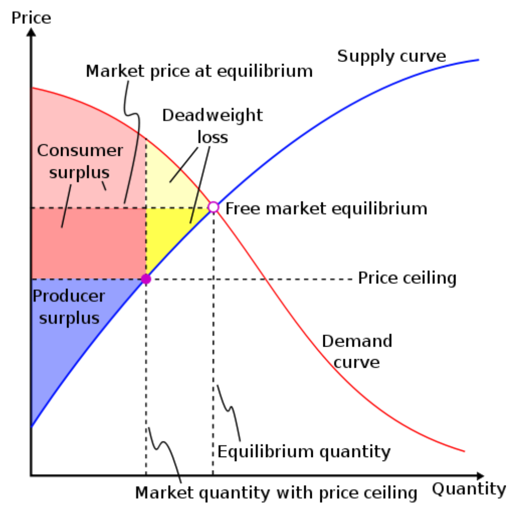
Games and Decisions

Market Structures

- the list of basic market structure types (seller-side types only):

	Number of sellers	Seller entry barriers	Deadweight loss
Perfect competition	Many	No	None
Monopolistic competition	Many	No	None
Oligopoly	Few	Yes	Medium
Monopoly	One	Yes	High



Collusive vs. Non-Collusive Oligopolies

- note: oligopoly differs from monopoly (allocation-wise) only if there's no *collusion*
  - collusion: a largely illegal form of cooperation amongst the sellers that includes price fixing, market division, total industry output control, profit division, etc.
  - controlled by competition/anti-trust laws
  - well-known collusion cases: OPEC, telecommunication, drugs, sports, chip dumping (poker)

- game-theoretical models:
  - cooperative setting (*collusive oligopoly*) → coalition theory
    - games in the *characteristic-function form*
  - non-cooperative setting (*competitive*, *non-collusive oligopoly*) → normal form game analysis
    - NE's etc.; however, matrices can't typically be used for payoffs

Oligopoly – Model Specification

- to make the analysis simple, we'll make several assumptions:

1. *single-product model*: oligopolists produce a single type of *homogenous* product
2. *one strategic variable*: firms decide about prices *or* output levels
3. *static model*: single-period analysis only
   - in dynamic models, there are more diverse strategic options: elimination of competitors even with contemporary losses etc.
4. *single objective*: all firms maximize their individual profit

Three basic non-cooperative oligopoly models:

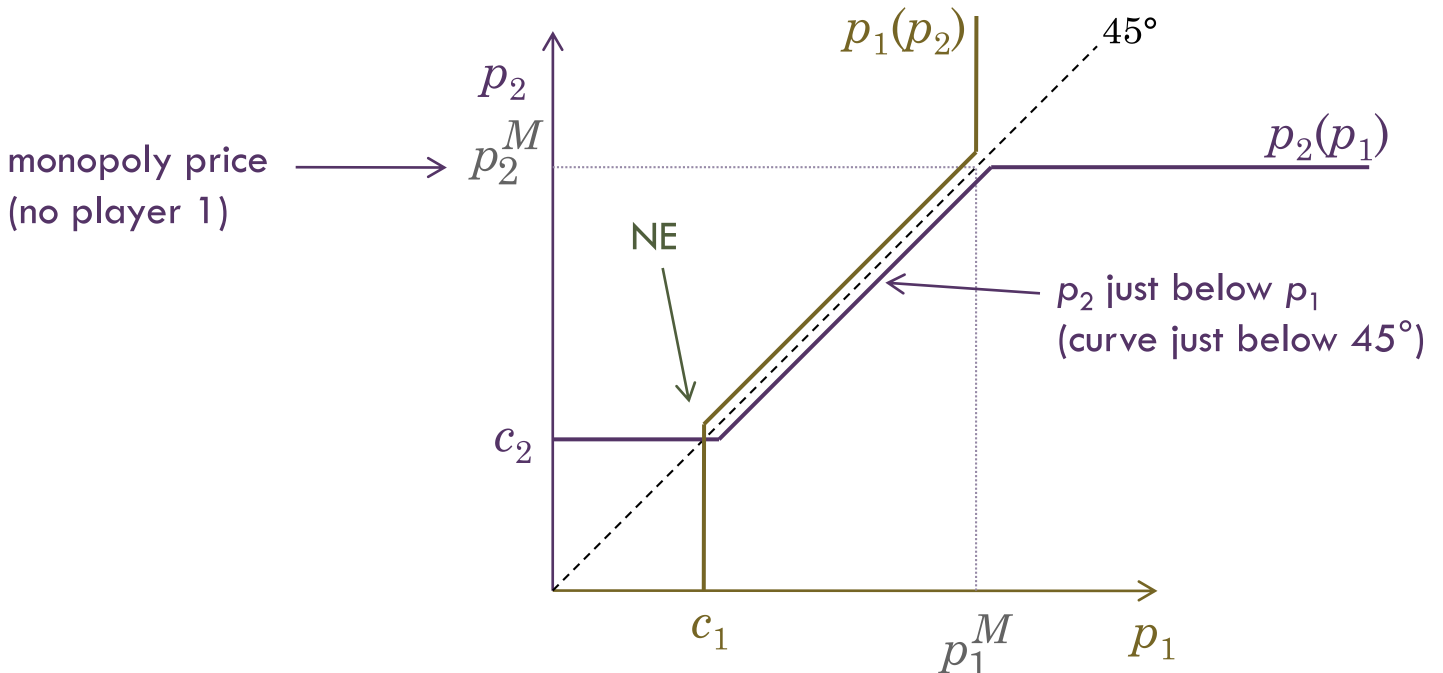
- *Bertrand* oligopoly – firms simultaneously choose prices
- *Cournot* oligopoly – firms simultaneously choose quantities
- *Stackelberg* oligopoly – firms choose quantities sequentially
  - *note: sequential-move games are typically not modelled as normal-form games. Instead, we use the extensive-form approach (not this lecture).*

Bertrand Duopoly

- Bertrand duopoly (2 oligopolists only) – model notation:
  - market demand function: $q = D(p)$
  - prices charged by the players: p_1, p_2
  - resulting quantities: q_1, q_2
  - unit costs: c_1, c_2 (for simplicity: $AC = MC = c$)
- homogenous product → lower price attracts all the consumers
  - $p_1 < p_2 \rightarrow q_1 = D(p_1), \quad q_2 = 0$
  - $p_1 > p_2 \rightarrow q_1 = 0, \quad q_2 = D(p_2)$
  - $p_1 = p_2 \rightarrow$ equal market share, $q_1 = q_2 = ½\, D(p_1) = ½\, D(p_2)$
- as long as the prices are higher than c_1 and c_2, both oligopolists tend to push prices down (below the other player's price)
  - imagine the prices are equal and above c_1; by lowering the price just slightly, player 1 can gain the whole market (if p_2 stays the same)
  - best response of player 1 to p_2 is to choose $p_1 = p_2 - \varepsilon$ ("*just below*" p_2) (until the prices reach c_1 → player 1 suffers a loss below)

Bertrand Duopoly (cont'd)

- NE depends on the *MC* of the players:
  - $c_1 = c_2 \quad \rightarrow \quad p_1^* = p_2^* = c_1 = c_2$ → zero economic profit for both
  - $c_1 < c_2 \quad \rightarrow \quad p_1^* = c_2 - \varepsilon$ → player 1 wins all, positive profit
  - $c_1 > c_2 \quad \rightarrow \quad p_2^* = c_1 - \varepsilon$ → player 2 wins all, positive profit
- more precisely: graphical best-response analysis – reaction curves:



Bertrand Duopoly (cont'd)

→ price competition leads to fairly efficient allocation

Critique of the Bertrand model (or, when Bertrand model fails to work)

- capacity constraints of production
  - e.g., consider the $c_1 < c_2$ situation: if player 1 can't supply enough for the whole market, player 2 can still charge p_2 above c_2 and attract some customers (and achieve a positive profit)
  - if $c_1 = c_2$ and neither player can supply to all customers, either player can raise the output price above c
- lack of product homogeneity (homogeneity disputable in most cases)
- transaction/transportation costs:
  - may differ for the specific customer–firm interactions
    - e.g., shops at both ends of a street: people tend to pick the closer one
    - if transportation costs are accounted for, the consumer expenditures vary even if prices are equal

Cournot Oligopoly – Formal Treatment

- model type – normal-form game with the following elements:
  - **list of firms**: $1, 2, \ldots, N$
  - **strategy spaces**: $X_1, X_2, \ldots, X_N$
    - potential quantities: typically intervals like [0,1000] → *infinite* alternatives!
    - the output level produced by i^{th} player (the strategy adopted) is denoted x_i
    - a **strategy profile** is an N-tuple: (where $x_i \in X_i$) $(x_1, x_2, \ldots, x_N)$
  - **cost functions**: $c_1(x_1),\, c_2(x_2), \ldots,\, c_N(x_N)$
    - total cost as the function of output level
  - **price function** (or **inverse demand** function): $p = f(x_1 + x_2 + \ldots + x_N)$
    - i.e., market price is the function of *total industry output*
- **profit** of i^{th} firm: $\pi_i(x_1, \ldots, x_N) = TR_i - TC_i = x_i \times f(x_1 + \ldots + x_N) - c_i(x_i)$

Nash Equilibrium in Cournot Oligopoly

- **mathematical definition**:

  A strategy profile $(x_1^*, x_2^*, \ldots, x_N^*)$ *is a* NE *if for all* $i = 1,\ldots,N$

$$\pi_i(x_1^*, x_2^*, \ldots, x_i, \ldots, x_N^*) \le \pi_i(x_1^*, x_2^*, \ldots, x_i^*, \ldots, x_N^*)$$

  holds for all $x_i \in X_i$.

- finding the NE: best-response approach (again)
  - NE: the strategies have to be the best responses to one another
  - best-response functions:
    - for player 1: $r_1(x_2,\ldots,x_N)$ is the best-response x_1 chosen by player 1, given that the other player's quantities are $x_2,\ldots,x_N$
    - mathematically: $r_1(x_2,\ldots x_N) = \arg\max_{x_1 \in X_1} \pi_i(x_1, x_2, \ldots, x_N)$
    - NE: $x_i^* = r_i(x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_N^*)$ for $i = 1,\ldots,N$

Example 1: Cournot Duopoly

- price function: $p = f(x_1 + x_2) = 100 - (x_1 + x_2)$
- other characteristics: $X_1 = [0, +\infty)$ $\quad c_1(x_1) = 150 + 12x_1$

  $X_2 = [0, +\infty)$ $\quad c_2(x_2) = x_2^2$
- profit functions:

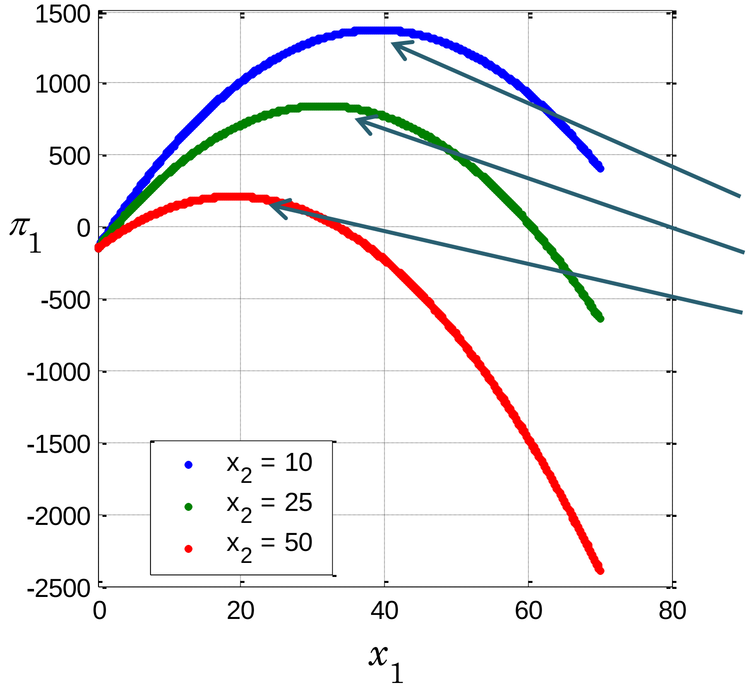
$$\begin{aligned}\pi_1(x_1, x_2) &= x_1 \times f(x_1 + x_2) - c_1(x_1) = \\ &= x_1 \times [100 - (x_1 + x_2)] - (150 + 12x_1) = \\ &= 100x_1 - x_1^2 - x_1x_2 - 150 - 12x1 = \\ &= 88x_1 - x_1^2 - x_1x_2 - 150\end{aligned}$$

$$\pi_2(x_1, x_2) = \ldots = 100x_2 - 2x_2^2 - x_1x_2$$

- best response of player 1 to x_2: profit-maximizing (π_1-maximizing) value of x_1 for the given x_2

Example 1: Cournot Duopoly (cont'd)

- profit of player 1 for three different levels of x_2:

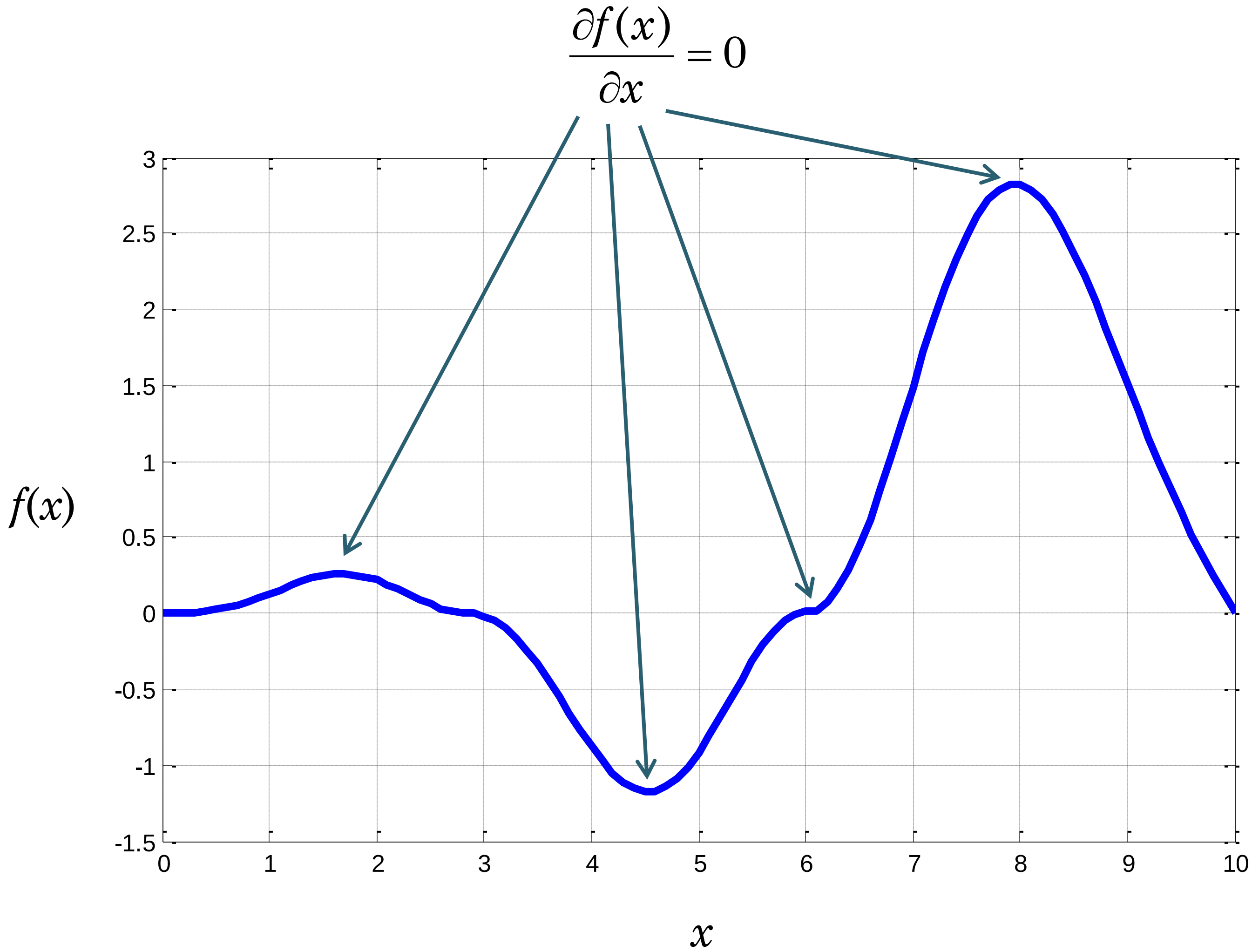


best response to $x_2 = 10$ is $x_1 \approx 40$,
best response to $x_2 = 25$ is $x_1 \approx 33$,
best response to $x_2 = 50$ is $x_1 \approx 20$,

- best response for an arbitrary level of x_2: as the function $\pi_1(x_1,x_2)$ is *strictly concave* in x_1 for any x_2, we can use the first-order condition for a local extreme

$$\frac{\partial \pi_1(x_1,x_2)}{\partial x_1} = 88 - 2x_1 - x_2 \overset{!}{=} 0$$

Note: first order conditions are generally not *sufficient* for a maximum, only *necessary* conditions for *extreme* (but: concave function → global maximum)



Example 1: Cournot Duopoly (cont'd)

- we can also write the result in terms of the *reaction function* r_1:

$$\frac{\partial \pi_1(x_1,x_2)}{\partial x_1} = 88 - 2x_1 - x_2 \overset{!}{=} 0 \quad \Rightarrow \quad x_1 = r_1(x_2) = 44 - \frac{x_2}{2}$$

- similarly, for player 2, we obtain:

$$\frac{\partial \pi_2(x_1,x_2)}{\partial x_2} = 100 - x_1 - 4x_2 \overset{!}{=} 0 \quad \Rightarrow \quad x_2 = r_2(x_1) = 25 - \frac{x_1}{4}$$

- altogether, we have 2 linear equations; for NE strategies, both have to hold at the same time → in order to find the NE, we just need to solve

$$\begin{aligned} 88 - 2x_1{}^* - x_2{}^* &= 0 \\ 100 - x_1{}^* - 4x_2{}^* &= 0 \end{aligned} \quad \text{or} \quad \begin{aligned} x_1{}^* &= r_1(x_2{}^*) \\ x_2{}^* &= r_2(x_1{}^*) \end{aligned} \quad \Rightarrow \quad \begin{aligned} x_1{}^* &= 36 \\ x_2{}^* &= 16 \end{aligned}$$

Question:

What are the equilibrium profits and price?

Collusive Oligopoly

- *model framework*: as in case of Cournot oligopoly, only that players can form coalitions
- **coalition**: a group of firms that coordinate output levels and redistribute profits
- **grand coalition**: the coalition of all oligopolists, $Q = \{1,2,\ldots,N\}$
  - other coalitions are denoted by $K, L, \ldots$
  - a "single-firm coalition" is still called a coalition, e.g. $K = \{2\}$, and so is the "empty coalition" $\{\emptyset\}$

> Question:
>
> How many different coalitions can be formed with *N* firms?

- **characteristic function** (of the oligopoly): a function $v(K)$ that assigns to any coalition K the maximum attainable total profit of K
  - *payoff function*: single player, individual payoff, for a given strategy profile
  - *characteristic function*: coalition, sum of members' profits, max. attainable

Collusive Oligopoly (cont'd)

- characteristic function for grand coalition:

$$v(Q) = \max_{(x_1,\ldots,x_N)} \sum_{i=1}^{N} \pi_i(x_1,\ldots,x_N)$$

- characteristic function for other coalitions: profit of coalition members depends on the quantity chosen by non-members

→ what will the other players do? (Generally, difficult to say.)

1. **minimax characteristic function**: assume non-members supply as much as they can (up to their capacity constraints)

2. **equilibrium characteristic function**: assume the other players choose the NE quantities

- characteristic function for an arbitrary coalition:

$$v(K) = \max_{(x_i)_{i\in K}} \sum\nolimits_{i\in K} \pi_i(x_1,\ldots,x_N)$$

Example 2: Collusive Duopoly

- consider the same duopoly as in example 1, only with capacity constraints:
  - price function: $p = f(x_1 + x_2) = 100 - (x_1 + x_2)$
  - other characteristics: $X_1 = [0,40]$ $\quad c_1(x_1) = 150 + 12x_1$
    $X_2 = [0,20]$ $\quad c_2(x_2) = x_2^2$
  - profit functions: $\pi_1(x_1, x_2) = 88x_1 - x_1^2 - x_1x_2 - 150$
    $\pi_2(x_1, x_2) = 100x_2 - 2x_2^2 - x_1x_2$
- first, we'll find the *equilibrium characteristic function*:
  - we already know the NE values: $x_1^* = 36$ $\quad \pi_1^* = 1146$
    $x_2^* = 16$ $\quad \pi_2^* = 512$
  - immediately, we have: $v(1) = \max_{x_1 \in X_1} \pi_1(x_1, 16) = \pi_1(36,16) = 1146$
    $v(2) = \max_{x_2 \in X_2} \pi_2(36, x_2) = \pi_2(36,16) = 512$

Example 2: Collusive Duopoly (cont'd)

- for grand coalition $Q = \{1,2\}$, we obtain:

$$v(1,2) = \max_{(x_1,x_2)} \pi_1(x_1,x_2) + \pi_2(x_1,x_2) =$$

$$= \max_{(x_1,x_2)} 88x_1 + 100x_2 - x_1^2 - 2x_2^2 - 2x_1x_2 - 150$$

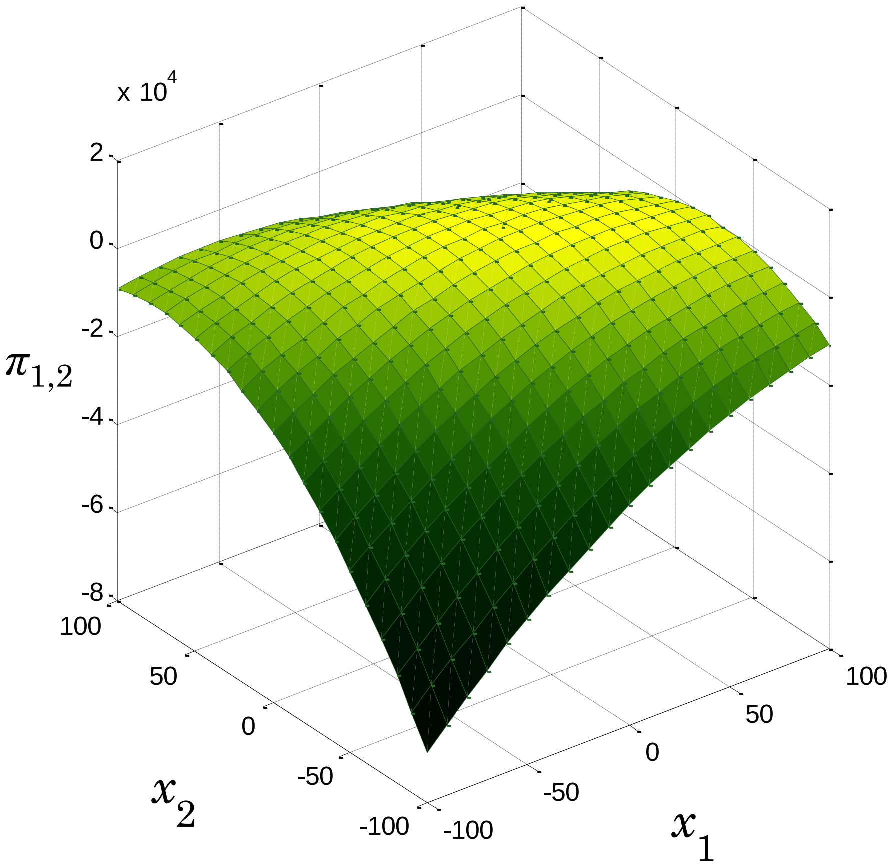
- function of *two variables* now, but still concave (see next slide)

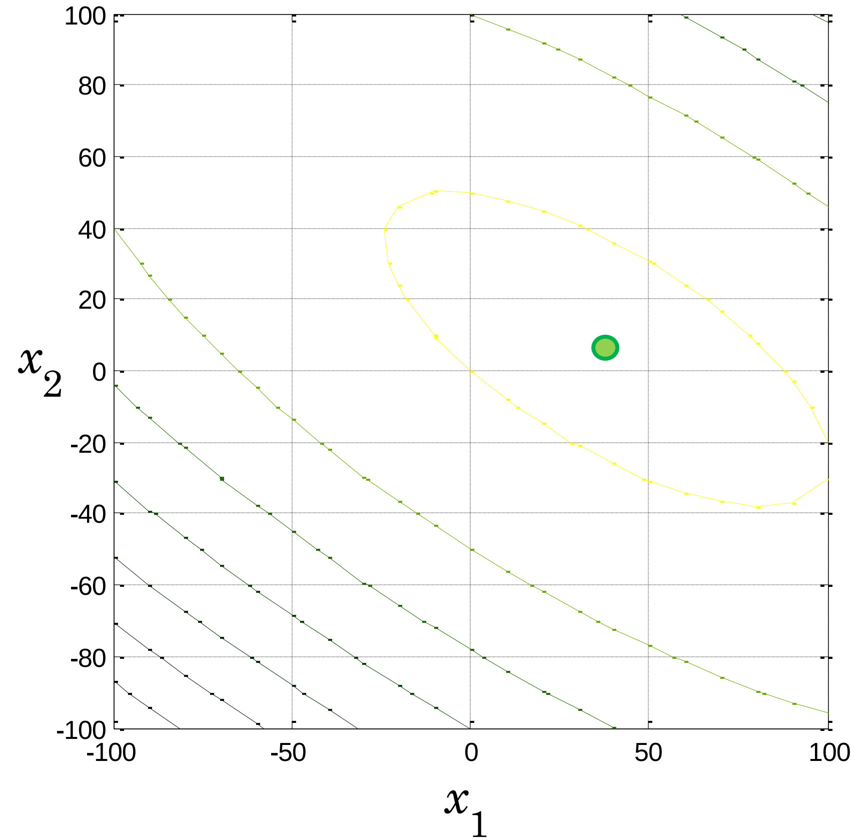
→ *first-order conditions* (both partial derivatives equal zero)

$$\left.\begin{array}{l} \dfrac{\partial \pi_{1,2}(x_1,x_2)}{\partial x_1} = 88 - 2x_1 - 2x_2 \overset{!}{=} 0 \\ \dfrac{\partial \pi_{1,2}(x_1,x_2)}{\partial x_2} = 100 - 2x_1 - 4x_2 \overset{!}{=} 0 \end{array}\right. \Rightarrow \left.\begin{array}{l} x_1^{\text{opt}} = 38 \\ x_2^{\text{opt}} = 6 \end{array}\right\} \pi_{1,2}^{\text{opt}} = 1822$$

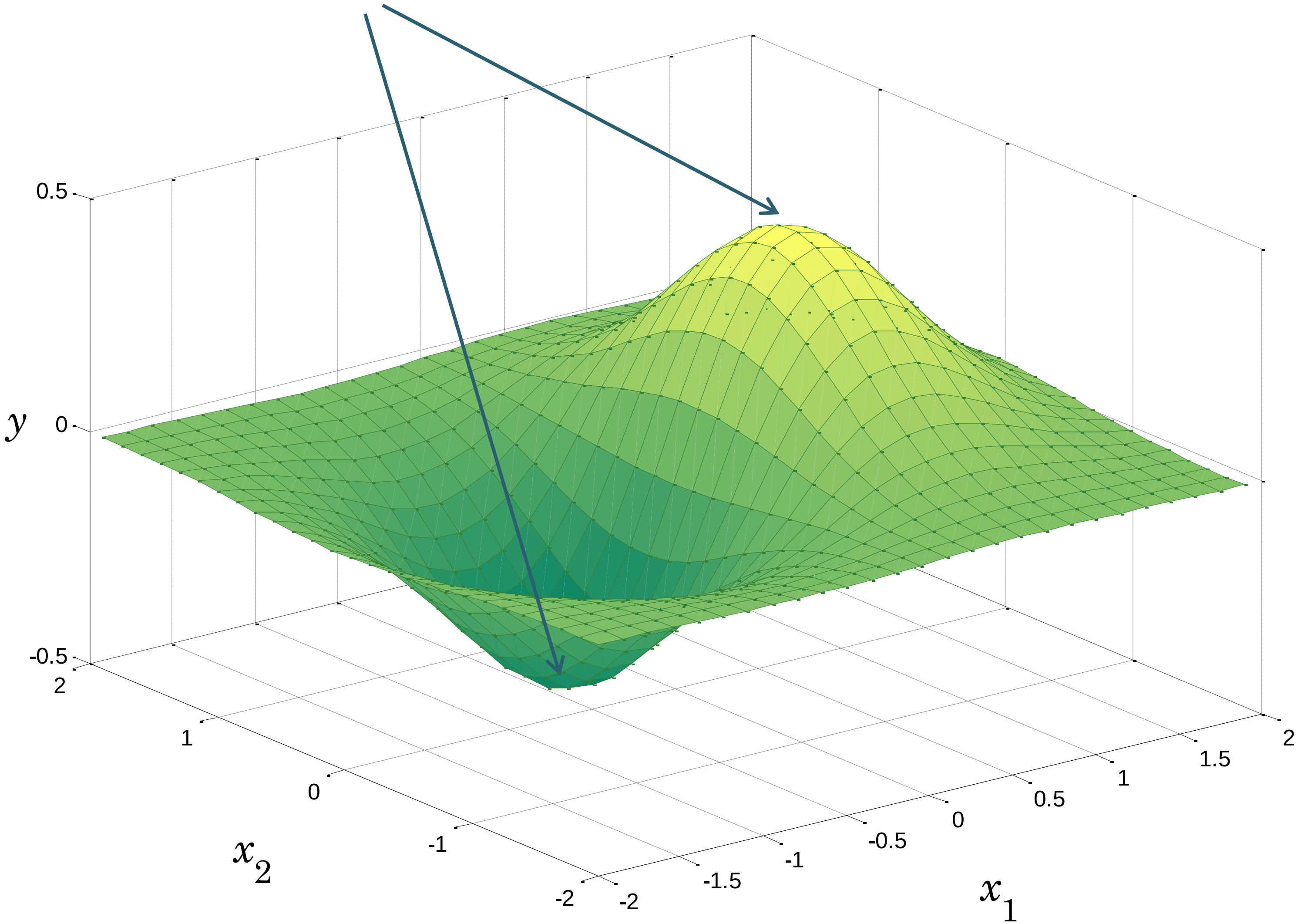
- $v(Q) = v(1,2) = 1822$

Example 2: Collusive Duopoly (cont'd)





first-order conditions: necessary conditions for local extremes (*not* sufficient, *not* for maxima only!)



Example 2: Collusive Duopoly (cont'd)

- the complete *equilibrium characteristic function* is as follows:

$$v(\emptyset) = 0$$
$$v(1) = 1146$$
$$v(2) = 512$$
$$v(1,2) = 1822$$

- *minimax characteristic function*:
  - $v(\emptyset)$ and $v(1,2)$ are the same as in the equilibrium char. function
  - for $v(1)$ and $v(2)$, we calculate the players' profits under the condition that the other player produces up to his/her capacity constraint:

$$v(1) = \max_{x_1 \in X_1} \pi_1(x_1, 20) = \max_{x_1 \in X_1} 68x1 - x_1^2 - 150 = \pi_1(34, 20) = 1006$$

$$v(2) = \max_{x_2 \in X_2} \pi_2(40, x_2) = \max_{x_2 \in X_2} 60x_2 - 2x_2^2 = \pi_2(40,15) = 450$$

Example 2: Collusive Duopoly (cont'd)

- a comparison of the two characteristic functions:

	equilibrium	minimax
$v(\emptyset)$	0	0
$v(1)$	1146	1006
$v(2)$	512	450
$v(1,2)$	1822	1822
$v(1,2) - v(1) - v(2)$	164	366

- core of the oligopoly: a division of payoffs a_1, a_2 such that

$$\begin{aligned} a_1 + a_2 &= 1822, \\ a_1 &\geq 1146, \\ a_2 &\geq 512, \end{aligned} \quad \text{or} \quad \begin{aligned} a_1 + a_2 &= 1822, \\ a_1 &\geq 1006, \\ a_2 &\geq 450. \end{aligned}$$

Lecture 6: Oligopoly

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