## LECTURE 3: <br> Mixed Strategies

Jan Zouhar Games and Decisions

## Matrix Games (revision)

$\square$ a special case of zero-sum games:

- a finite set of agents:
- strategy spaces (finite):
- strategy profile:
- payoff functions:
$Z_{1}(x, y), Z_{2}(x, y)$
- zero-sum payoffs: $Z_{1}(x, y)+Z_{2}(x, y)=0$
$\square$ payoffs written in a matrix, typically denoted by $\boldsymbol{A}$ :

$$
\boldsymbol{A}=\left(a_{i j}\right)_{\substack{i=1, \ldots, m \\
j=1, \ldots, n}}=\left[\begin{array}{cccc}
a_{11} & a_{12} & \ldots & a_{1 n} \\
a_{21} & a_{22} & \ldots & a_{2 n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m 1} & a_{m 2} & \ldots & a_{m n}
\end{array}\right]
$$

$\square a_{i j}=$ the payoff of player 1 for strategy profile $(i, j)$ (i.e., player 1 picks $i$ th strategy and player 2 picks $j$ th)

## Nash Equilibrium in Matrix Games (revision)

$\square$ mathematical definition:
A strategy profile ( $x^{*}, y^{*}$ ) with the property that

$$
Z_{1}\left(x, y^{*}\right) \leq Z_{1}\left(x^{*}, y^{*}\right) \leq Z_{1}\left(x^{*}, y\right)
$$

for all $x \in X$ and $y \in Y$ is a NE.

Inequality from the definition above explained:


## Matrix Games with No Pure Strategy NE

Rock, Paper, Scissors

|  | Player 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $1 \backslash 2$ | $\mathbf{R}$ | $\mathbf{P}$ |  |
|  | R | 0 | -1 |  |
| Player 1 | P | 0 | 0 |  |



## Matrix Games with No Pure Strategy NE (cont'd)

## Matching pennies game

$\square$ both players secretly turn a coin to heads/tails
$\square \quad$ if both heads or both tails, player 1 pays $\$ 1$, otherwise player 2 pays $\$ 1$

Player 2

|  | $1 \backslash 2$ | Heads | Tails |
| :---: | :---: | :---: | :---: |
|  | Heads | -1 | 1 |
| Player 1 | 1 |  |  |
|  | Tails | 1 | -1 |

## Matrix Games with No Pure Strategy NE (cont'd)

Penalty kicks (a modified version of matching pennies)
$\square$ Kick vs. Goalkeeper

- strategies: Right/Left
$\square$ payoffs: scoring probabilities

Goalkeeper

|  | $1 \backslash 2$ | Right | Left |
| :---: | :---: | :---: | :---: |
| Kick | Right | 0.5 | 0.9 |
|  | Left | 0.9 | 0.5 |



## Matrix Games with No Pure Strategy NE

- stability in NE: even if he knows his opponent is playing the NE strategy, player 1 has no incentive to deviate from NE (the same goes for player 2)
$\square$ matching pennies:
- if player one knows player 2 is playing Heads, he can win by playing Tails and vice versa
$\rightarrow$ neither Heads nor Tails can be the NE strategy
$\square$ still, there is a rational way to play the game:
- each time you play the game, toss the coin and let it decide the strategy on itself
- even if your opponent knows your strategy, she can't take advantage of that (compare with $R P S$ game)
- tossing a coin means applying a mixed strategy


## Pure Vs. Mixed Strategies in Matrix Games

$\square$ switch from pure strategies to mixed strategies:

- pure strategy: the player decides for a particular strategy (i.e., picks a certain row or column)
- mixed strategy:
- the player decides about the probabilities of the alternative strategies (sum of the probabilities $=1$ )
- when the decisive moment comes, he/she makes a random selection of the strategy with the stated probabilities
$\square$ allowing for mixed strategies in a matrix game: sometimes called a mixed extension of a matrix game
$\square$ notation: mixed strategies $=$ column vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, ith element represents the probability of $i$ th row/column being picked


## Mixed Strategies: An Example

- matching pennies game
- player 1 always chooses Heads
- player 2 tosses a coin
$\rightarrow$ the resulting mixed strategies:

|  | Player 2 |  |  |
| :---: | :---: | :---: | :---: |
|  | $1 \backslash 2$ | Heads | Tails |
| Player 1 | Heads | -1 | 1 |
|  | Tails | 1 | -1 |

Heads probabilities


Tails probabilities
$\square$ note: $\boldsymbol{x}$ actually is a pure strategy here

## Payoffs with Mixed Strategies

$\square \quad$ when using mixed strategies, the payoffs for the individual players become random variables

- the possible values are stated in the game's matrix
- the final outcome depends on the strategies eventually picked by the individual players
$\square$ in order to be able to treat the payoffs resulting from a combination of mixed strategies as a single number, we use the concept of expected payoff


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## Expected value:

Let $Z$ be a discrete-valued random variable with possible values in
$\Omega$. The expected value of $Z$ is:

$$
\mathrm{E} Z=\sum_{z \in \Omega} z \cdot \operatorname{Pr}\{Z=z\}
$$

## Expected Payoffs: An Example

$\square$ consider matching pennies again, with $\boldsymbol{x}=\left[\begin{array}{ll}1 & 0\end{array}\right]^{\top}, \boldsymbol{y}=\left[\begin{array}{ll}0.5 & 0.5\end{array}\right]^{\top}$

- the expected payoff of player one is:

$$
\begin{aligned}
\mathrm{E} Z_{1} & =\left\{\begin{array}{cc}
-1 \times \operatorname{Pr}(H, H) & +1 \times \operatorname{Pr}(H, T) \\
+1 \times \operatorname{Pr}(T, H) & -1 \times \operatorname{Pr}(T, T)
\end{array}\right\} \\
& =\left\{\begin{array}{cc}
-1 \times(1 \times 0.5) & +1 \times(1 \times 0.5) \\
+1 \times(0 \times 0.5) & -1 \times(0 \times 0.5)
\end{array}\right\} \\
& =\left\{\begin{array}{cc}
-0.5 & +0.5 \\
+0 & +0
\end{array}\right\}=0
\end{aligned}
$$

- the expected value of player 2 is:

$$
\mathrm{E} Z_{2}=-\mathrm{E} Z_{1}=0
$$

|  |  | 0.5 | 0.5 |
| :---: | :---: | :---: | :---: |
|  | $1 \backslash 2$ | Heads | Tails |
| 1 | Heads | -1 | 1 |
| 0 | Tails | 1 | -1 |

## Expected Payoffs: A Generalization

- consider a general $2 \times 2$ matrix game with matrix $\boldsymbol{A}$ and mixed strategies $\boldsymbol{x}$ and $\boldsymbol{y}$ :

$$
\boldsymbol{x}=\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right], \quad \boldsymbol{y}=\left[\begin{array}{l}
y_{1} \\
y_{2}
\end{array}\right], \quad \boldsymbol{A}=\left[\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right] . \quad \begin{array}{c|cc} 
& y_{1} & y_{2} \\
\hline x_{1} & a_{11} & a_{12} \\
x_{2} & a_{21} & a_{22}
\end{array}
$$

$\square$ the expected payoff of player 1:

$$
\begin{aligned}
\mathrm{E} Z_{1}= & x_{1} a_{11} y_{1}+x_{1} a_{12} y_{2} \\
& +x_{2} a_{21} y_{1}+x_{2} a_{22} y_{2}=\sum_{i} \sum_{j} x_{i} a_{i j} y_{j}=\boldsymbol{x}^{\top} \boldsymbol{A} \boldsymbol{y}
\end{aligned}
$$

- the last two expressions hold for the general case of $m \times n$ matrix games


## Expected Payoffs: A Generalization (cont'd)

Expected payoffs in an $m \times n$ matrix game with mixed strategies:

$$
\mathrm{E} Z_{1}=\sum_{i=1}^{m} \sum_{j=1}^{n} x_{i} a_{i j} y_{j}=\boldsymbol{x}^{\top} \boldsymbol{A} \boldsymbol{y}=-\mathrm{E} Z_{2}
$$



## Expected payoffs: Exercise 1

$\square$ calculate the expected payoff for both players in the penalty kicks game for the following mixed strategies of the two players:
a) $\boldsymbol{x}=\left[\begin{array}{ll}1 & 0\end{array}\right]^{\top}, \quad y=\left[\begin{array}{ll}0.5 & 0.5\end{array}\right]^{\top}$.
b) $\boldsymbol{x}=\left[\begin{array}{ll}0.5 & 0.5\end{array}\right]^{\top}, \quad y=\left[\begin{array}{ll}0.5 & 0.5\end{array}\right]^{\top}$.
c) $\boldsymbol{x}=\left[\begin{array}{ll}0 & 1\end{array}\right]^{\top}, \quad \boldsymbol{y}=\left[\begin{array}{ll}0.5 & 0.5\end{array}\right]^{\top}$.

Goalkeeper

|  | $1 \backslash 2$ | Right | Left |
| :---: | :---: | :---: | :---: |
| Kick | Right | 0.5 | 0.9 |
|  | Left | 0.9 | 0.5 |

## Nash Equilibrium with Mixed Strategies

$\square$ overall concept: the same as with pure strategies
"NE is such a combination of strategies that neither of the players can increase their payoff by choosing a different strategy."
"NE is a solution with the property that whoever of the players chooses some other strategy, he or she will not increase his or her payoff."

- mathematical definition:

NE is a combination of (mixed) strategies $\boldsymbol{x}^{*}$ and $\boldsymbol{y}^{*}$ with the property that

$$
\boldsymbol{x}^{\top} \boldsymbol{A} \boldsymbol{y}^{*} \leq \boldsymbol{x}^{* \top} \boldsymbol{A} \boldsymbol{y}^{*} \leq \boldsymbol{x}^{* \top} \boldsymbol{A} \boldsymbol{y}
$$

for all mixed strategies $\boldsymbol{x}$ and $\boldsymbol{y}$.
$\square$ value of the game: player 1's expected payoff at NE $\left(\boldsymbol{x}^{* \top} \boldsymbol{A} \boldsymbol{y}^{*}\right)$

## Nash Equilibrium with Mixed Strategies

## Inequality from the NE definition explained:

Can be rewritten as:

$$
\mathrm{E} Z_{1}\left(\boldsymbol{x}, \boldsymbol{y}^{*}\right) \leq \mathrm{E} Z_{1}\left(\boldsymbol{x}^{*}, \boldsymbol{y}^{*}\right),
$$

which means: If player 1 deviates from NE, his/her expected payoff will not increase

Can be rewritten as:

$$
-\mathrm{E} Z_{2}\left(\boldsymbol{x}^{*}, \boldsymbol{y}^{*}\right) \leq-\mathrm{E} Z_{2}\left(\boldsymbol{x}^{*}, \boldsymbol{y}\right), \quad \text { or } \quad \mathrm{E} Z_{2}\left(\boldsymbol{x}^{*}, \boldsymbol{y}\right) \leq \mathrm{E} Z_{2}\left(\boldsymbol{x}^{*}, \boldsymbol{y}^{*}\right) \text {, }
$$

which means: If player 2 deviates from $N E$, his/her expected payoff will not increase

## Basic Theorem on Matrix Games

$\square \quad$ it is easily seen that a pure-strategy NE is also a mixed-strategy NE; therefore, if a matrix game has a NE in pure strategies, it has a NE in mixed strategies as well
$\square$ what happens in the are no pure-strategy NE? As JoHn von NEUMANN proved in 1928, even if a matrix game has no NE in pure strategies (i.e., no saddle point of the payoff matrix), it still has a NE in mixed strategies (always)

- Basic Theorem on Matrix Games:

For any matrix $\boldsymbol{A}$ there exist mixed strategies $\boldsymbol{x}^{*}$ and $\boldsymbol{y}^{*}$ such that

$$
\boldsymbol{x}^{\top} \boldsymbol{A} \boldsymbol{y}^{*} \leq \boldsymbol{x}^{\star \top} \boldsymbol{A} \boldsymbol{y}^{*} \leq \boldsymbol{x}^{\star \top} \boldsymbol{A} \boldsymbol{y}
$$

for all mixed strategies $\boldsymbol{x}$ and $\boldsymbol{y}$.

## Finding Mixed-Strategy NE's

$\square$ Basic Theorem tells us that every matrix game has a mixed-strategy solution
$\square$ two possible approaches of finding the solution depending on the size of the matrix $\boldsymbol{A}$ :
a) "small" $-2 \times n$ and $m \times 2$ matrices:

- simple
- graphical solution
b) "general" $-m \times n$ matrices:
- a bit more complicated
- linear programming
- can be solved in MS Excel (or other SW)


## Graphical Solution - $2 \times 2$ Matrices

$\square \quad 2 \times 2$ matrix $\rightarrow$ players actually choose only 1 probability:

$$
x_{2}=1-x_{1}, \quad y_{2}=1-y_{1}
$$

$\square$ player 1's decision: $x_{1}$ (probability of $T$ )

- expected payoffs:
- player 2 picks $L: \quad \mathrm{E} Z_{1}=1 \times x_{1}+11 \times\left(1-x_{1}\right)=11-10 x_{1}$
- player 2 picks $R: \quad \mathrm{E} Z_{1}=7 \times x_{1}+2 \times\left(1-x_{1}\right)=2+5 x_{1}$
- player 2 chooses a mixed strategy: anything in between (more precisely: a convex combination of the two)



## Graphical Solution $-2 \times 2$ Matrices

player 2's best response to possible $x_{1}$ values

player 1 maximizes $E Z_{1}$ while taking player 2's best response into account

## Graphical Solution - $2 \times 2$ Matrices

$\square$ player 2's decision: $y_{1}$ (probability of $L$ )

- expected payoffs:
- player 1 picks $T: \quad \mathrm{E} Z_{1}=1 \times y_{1}+7 \times\left(1-y_{1}\right) \quad=7-6 y_{1}$
- player 1 picks $B: \quad \mathrm{E} Z_{1}=11 \times y_{1}+2 \times\left(1-y_{1}\right)=2+9 y_{1}$
- player 1 chooses a mixed strategy: anything in between (more precisely: a convex combination of the two)

|  | Player 2 |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathbf{1} \backslash \mathbf{2}$ | L | R |
| Player 1 | T | $\mathbf{1}$ | 7 |
|  | B | $\mathbf{1 1}$ | $\mathbf{2}$ |


|  | $y_{1}$ | $1-y_{1}$ |
| :---: | :---: | :---: |
| $x_{1}$ | $a_{11}$ | $a_{12}$ |
| $1-x_{1}$ | $a_{21}$ | $a_{22}$ |

## Graphical Solution $-2 \times 2$ Matrices

player l's best response to possible $y_{1}$ values

player 2 minimizes $E Z_{1}$ while taking player l's best response into account

## Graphical Solution - $2 \times 2$ Matrices

$\square$ finding $x_{1}{ }^{*}$ and $y_{1}{ }^{*}$ :

- graphical solution: intersection of two lines
- numerically: system of two linear equations
- for player 1:

$$
\left.\begin{array}{l}
\mathrm{E} Z_{1}=11-10 x_{1} \\
\mathrm{E} Z_{1}=2+5 x_{1}
\end{array}\right\} \quad \begin{aligned}
x_{1} & =3 / 5 \\
\mathrm{E} Z_{1} & =5
\end{aligned}
$$

- for player 2:

$$
\left.\begin{array}{l}
\mathrm{E} Z_{1}=7-6 y_{1} \\
\mathrm{E} Z_{1}=2+9 y_{1}
\end{array}\right\} \quad \begin{aligned}
y_{1} & =1 / 3 \\
\mathrm{E} Z_{1} & =5
\end{aligned}
$$

- equilibrium mixed strategies:

$$
\boldsymbol{x}^{*}=\left[\begin{array}{c}
\frac{3}{5} \\
\frac{2}{5}
\end{array}\right], \quad \boldsymbol{y}^{*}=\left[\begin{array}{c}
\frac{1}{3} \\
\frac{2}{3}
\end{array}\right] .
$$

$\square$ value of the game: $\boldsymbol{x}^{* \top} \boldsymbol{A} \boldsymbol{y}^{*}=5$

- represented by $\mathrm{E} Z_{1}$-value of the intersection in both plots


## Graphical Solution: Exercise 2

$\square$ penalty kicks game, kick better when aiming left
$\square$ use graphical solution to find NE strategies for both players

Goalkeeper

|  | $1 \backslash 2$ | Right | Left |
| :---: | :---: | :---: | :---: |
|  | Right | $30 \%$ | $80 \%$ |
| Kick | Left | $90 \%$ | $50 \%$ |

## Row and Column Differences Formula

$\square$ the solution of the system of two equations can be expressed easily using row and column differences:

$$
\begin{array}{ll}
r_{1}=a_{11}-a_{12} & c_{1}=a_{11}-a_{21} \\
r_{2}=a_{21}-a_{22} & c_{2}=a_{12}-a_{22}
\end{array}
$$

$\square$ NE strategies:

$$
x_{1}^{*}=\frac{r_{2}}{c_{2}-c_{1}}, \quad y_{1}^{*}=\frac{c_{2}}{r_{2}-r_{1}}
$$

Player 2

| Player 1 | $1 \backslash 2$ | L | R | -6 | $a_{11}$ | $a_{12}$ | $r_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | T | 1 | 7 |  |  |  |  |
|  | B | 11 | 2 | 9 | $a_{21}$ | $a_{22}$ | $r_{2}$ |
|  |  | -10 | 5 |  | $c_{1}$ | $c_{2}$ |  |

## Row and Column Differences Formula

$\square$ derivation of the formula for NE strategies from the systems of equations:

$$
\left.\begin{array}{ll}
L: & \mathrm{E} Z_{1}=a_{11} x_{1}+a_{21}\left(1-x_{1}\right)=a_{21}+c_{1} x_{1} \\
R: & \mathrm{E} Z_{1}=a_{12} x_{1}+a_{22}\left(1-x_{1}\right)=a_{22}+c_{2} x_{1}
\end{array}\right\} \quad x_{1}=\frac{r_{2}}{c_{2}-c_{1}}
$$

$$
\left.\begin{array}{ll}
T: & \mathrm{E} Z_{1}=a_{11} y_{1}+a_{12}\left(1-y_{1}\right)=a_{12}+r_{1} y_{1} \\
B: & \mathrm{E} Z_{1}=a_{21} y_{1}+a_{22}\left(1-y_{1}\right)=a_{21}+r_{2} y_{1}
\end{array}\right\} \quad y_{1}=\frac{c_{2}}{r_{2}-r_{1}}
$$

Player 2

| Player 1 | $1 \backslash 2$ | L | R | -6 | $a_{11} \quad a_{12}$ |  | $r_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | T | 1 | 7 |  |  |  |  |
|  | B | 11 | 2 | 9 | $a_{21}$ | $a_{22}$ | $r_{2}$ |
|  |  | -10 | 5 |  | $c_{1}$ | $c_{2}$ |  |

## Row and Column Differences: Exercise 3

$\square$ using row and column differences formula to find the NE strategies in the following game

Player 2


## Graphical Solution $-2 \times n$ Matrices

$\square$ a generalized version of $2 \times 2$ graphical solution
$\square$ start with player 1's plot

- determine the "active" best responses (strategies that can be played in a NE)
$\square$ consider only active best responses for player 2's plot
$\square$ for $m \times 2$ matrices, proceed similarly

Player 2

|  | $1 \backslash 2$ | L | M | R | Expected payoffs with $\mathbf{x}=\left[x_{1}, 1-x_{1}\right]^{\top}$ : |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Player 1 | T | 1 | 12 | 7 | L: $\mathrm{E} Z_{1}=1 \times x_{1}+11 \times\left(1-x_{1}\right)=11-10 x_{1}$ <br> M: $\mathrm{E} Z_{1}=12 \times x_{1}+0 \times\left(1-x_{1}\right)=12 x_{1}$ |
|  | B | 11 | 0 | 2 | $R: \mathrm{E} Z_{1}=7 \times x_{1}+2 \times\left(1-x_{1}\right)=2+5 x_{1}$ |

## Graphical Solution $-2 \times n$ Matrices



## Graphical Solution $-2 \times n$ Matrices



## Graphical Solution $-2 \times n$ Matrices

player l's best response to possible $y_{1}$ values

player 2 minimizes $E Z_{1}$ while taking player l's best response into account

## Finding NE - Linear Programming

$\square$ the construction of the algorithm and its explanation is actually presented in the proof of the Basic Theorem on Matrix Games (see the Games and Economic Decisions textbook, or next lecture)

## Linear programming:

Using linear programming methods, one can finding a maximum or minimum of a linear function of multiple variables on a set given by linear constraints:

## Finding NE - Linear Programming

- Step 1: If there is a negative element in the payoff matrix, make all elements of the matrix positive by adding the same positive number to all elements of the matrix. (This does changes the game, but only into a strategically equivalent one.)
- Step 2: Solve linear programming problem
maximize $p_{1}+p_{2}+\ldots+p_{n}$
subject to

$$
\begin{aligned}
& a_{11} p_{1}+a_{12} p_{2}+\ldots+a_{1 n} p_{n} \leq 1, \\
& a_{21} p_{1}+a_{22} p_{2}+\ldots+a_{2 n} p_{n} \leq 1, \\
& \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
& a_{m 1} p_{1}+a_{m 2} p_{2}+\ldots+a_{m n} p_{n} \leq 1, \\
& \quad p_{i} \geq 0, \quad i=1, \ldots, n .
\end{aligned}
$$

- Step 3: Divide the primal and dual solutions by the optimal value of the objective function:
- the primal solution determines the strategy of player 2.
$\square$ the dual solution determines the strategy of player 1 .


## Finding NE - Linear Programming

$\square$ note: if we use the symbol $\mathbf{1}_{n}$ to denote vector

$$
\left.\mathbf{1}_{n}=\left[\begin{array}{c}
1 \\
1 \\
\vdots \\
1
\end{array}\right]\right\} n \text { elements }
$$

we can simplify the LP problem from step 2 as

$$
\begin{aligned}
& \operatorname{maximize} z=\mathbf{1}_{n}^{\top} \boldsymbol{p} \\
& \text { subject to }
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{A p} & \leq \mathbf{1}_{m} \\
\boldsymbol{p} & \geq \mathbf{0} .
\end{aligned}
$$

## Using LP to Find NE: An Example

$\square$ we'll find mixed-strategy NE's in the following matrix game:

$\square$ Step 1: elimination of negative elements. We'll add a constant $c=2$ to all elements of the matrix (to get a strategically equivalent matrix game with non-negative elements.)

Player 2

|  | $1 \backslash 2$ | L | M | R |
| :---: | :---: | :---: | :---: | :---: |
| Player 1 | T | 3 | 3.5 | 1 |
|  | B | 2 | 6 | 4 |

## Using LP to Find NE: An Example

- Step 2: solve LP problem
maximize $p_{1}+p_{2}+p_{3}$
subject to

$$
\begin{aligned}
3 p_{1}+3.5 p_{2}+1 p_{3} & \leq 1, \\
2 p_{1}+6 p_{2}+4 p_{3} & \leq 1, \\
p_{1}, p_{2}, p_{3} & \geq 0 .
\end{aligned}
$$

- solve using MS Excel (to see how, download the NE_Solver.xls file from my website), optimal values are:
- objective function value: 0.4
- primal solution:
$\boldsymbol{p}=\left[\begin{array}{lll}0.3 & 0 & 0.1\end{array}\right]^{\top}$
- dual solution:

$$
\boldsymbol{q}=\left[\begin{array}{ll}
0.2 & 0.2
\end{array}\right]^{\top}
$$

$\square$ Step 3: the equilibrium strategies are:

- player 1: $\boldsymbol{x}^{*}=\boldsymbol{q} / 0.4=\left[\begin{array}{ll}0.5 & 0.5\end{array}\right]^{\top}$

口 player 2: $\boldsymbol{y}^{*}=\boldsymbol{p} / 0.4=\left[\begin{array}{lll}0.25 & 0 & 0.75\end{array}\right]^{\top}$

## Using LP to Find NE: An Example

$\square$ value of the game is

$$
\boldsymbol{x}^{* \top} \boldsymbol{A} \boldsymbol{y}^{*}=1 / \text { objective function }=1 / 0.4=2.5
$$

$\square$ value of the original game is
$1 /$ objective function $-c=1 / 0.4-2=2.5-2=0.5$
following slides: activating the Solver add-in in MS Excel 2007



## LECTURE 3: <br> Mixed Strategies

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