# LECTURE 1: INTRODUCTION

Jan Zouhar Games and Decisions

#### **Course Information**

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Jan Zouhar

### **Course Information**

- □ Course Requirements:
  - **50** points: **assignments** 
    - **5** problems, **10** points each
    - will appear on the website soon
    - due date: **December 11**
  - **50** points: **final written exam**
- Grading scale: standard ECTS points
  - **90 100** points: **excellent** (1)
  - **75 89** points: **very good** (2)
  - **60 74** points: **good** (3)
  - **0 59** points: **failed** (4)

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### **Course Information**

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- Recommended reading:
  - Lecture notes and presentations
  - MAŇAS, M., DLOUHÝ, M.: Games and Economic Decisions, Oeconomica, 2009
  - DLOUHÝ, M., FIALA, P.: Úvod do teorie her, VŠE, 2007 (in Czech)
  - Virtually any other book on game theory
  - Internet sources (e.g. <u>www.gametheory.net</u>, <u>www.wikipedia.org</u>)

### Game 1: Guess <sup>2</sup>/<sub>3</sub> of the Average

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- □ *Prize money*: **20 Kč** for the winner



- $\square$  Rules:
  - pick an integer between 0 and 100 and write it down onto a piece of paper
  - winner: the number closest to <sup>2</sup>/<sub>3</sub> of the average of all the guesses, without going over (i.e., the highest guess less than or equal to <sup>2</sup>/<sub>3</sub> of the average)
- □ *Empirical results*: winning guesses between **15** and **21**
- High guesses can mean two things:
  - 1. not understanding this is a strategic game
  - 2. believing the others do not understand this is a strategic game

### Game 2: An "Unfair" Auction

- Compete in an *English auction* ("open-outcry" type) for a 100 Kč banknote
- $\square$  Rules:
  - **starting price: 20 Kč**
  - buyers cry out increasing bids
  - the 100 Kč banknote sold to the highest bidder (at a price equal to his/her bid)
  - the second-highest bid is paid to the auctioneer without any compensation



Empirical results:

PAUL B. FARRELL: Lizards, rats & the investor's primitive brain.

### What Is a Game?

- many types of games: board games, card games, video games, field games (e.g. football)
- $\square$  we focus on games where:
  - □ there is more than one decision maker (*player*)
  - there is some choice of action where *strategy* matters
  - the game has one or more *outcomes*, e.g., someone wins, someone loses
  - the outcome depends on the strategies chosen by all players there is *strategic interaction*
  - the players are *rational* (are aware of the strategic interaction and act accordingly)
- $\rightarrow$  not games of pure chance, such as Bingo

#### Game vs. Decision Problem

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- □ *Example*: meal ordering: 4 people come to a restaurant
  - 1. every person pays for her own meal  $\rightarrow a$  decision problem
  - 2. everyone agrees to split the bill evenly  $\rightarrow a \ game$



- $\hfill\square$  when does a game become a decision problem?
  - players do not interact, no interdependencies between strategies
  - only 1 player (e.g., Solitaire)

### Strategy Interactions in Practice

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- □ International trade:
  - levels of imports, exports, prices depend not only on your own tariffs but also on tariffs of other countries
- □ *Production / market structure*:
  - price depends not only on your output but also on the output of your competitor
- $\square$  Labor:
  - promotions like tournaments: your chances depend not only on your effort but also on efforts of others
- Political economy:
  - who/what I vote for depends on what everyone else is voting for
- Description Public Goods:
  - my benefits from contributing to a public good depend on what everyone else contributes
- $\rightarrow$  suitable application areas of **game theory**

#### Game theory

#### **Definitions**:

- "...is a formal way to analyze interaction among a group of rational agents who behave strategically."
- "...can be defined as the study of mathematical models of conflict and cooperation between decision-makers."

"...offers insights into economic, political, or any social situation that involves multiple participants with different goals."

#### **a.k.a.** :

decision theory, conflict analysis, analysis of strategic behavior

#### Game Theory – Descriptive vs. Normative Use

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#### normative approach:

- **•** the analysis of rational behavior
- goal = find the optimal, or "most rational" form of behavior (or sometimes, the winning strategy)
- studies the way people should decide

#### descriptive approach:

- the analysis of the real-life behavior
  - sometimes, rational decisions are not obvious / people are not rational → normative approach gives us no clue (as in game 1)
- goal = study the way people really behave (for predictions etc.)
- studies the way people *decide in practice*
- normativity of a given game:
  - on average, 80% real-life players act according to the normative analysis  $\rightarrow$  *the game is 80% normative*

Games and Decisions

### Game Theory – Historical Milestones

- 1838: AUGUSTIN COURNOT, Researches into the Mathematical Principles of the Theory of Wealth
- 1913: ERNST ZERMELO existence of winning strategies in games like chess
- □ 1920s and 1930s: works of ÉMILE BOREL and JOHN VON NEUMANN
- □ 1944: J. VON NEUMANN and OSKAR MORGENSTERN, *Theory of Games* and *Economic Behaviour*
- □ 1950s: works of JOHN NASH
  - (film: A Beautiful Mind )
- $\Box$  Nobel prizes:
  - 1994: NASH, HARSANYI, SELTEN
  - **2005:** Aumann, Schelling

# BEAUTIFUL MIND

"An even greater gift is to discover a beautiful heart."

RUSSELL CROWE ~ ED HARRIS ~ JENNIFER CONNELLY

 $\uparrow \quad a film about John Nash \qquad \uparrow \\ \rightarrow \quad John Nash (a recent photo) \quad -$ 



### Game Theory – Applications

- $\square$  Economics and business
  - oligopolies, market structure, auctions, bargaining, fair division
- Political science
  - voting systems, coalition formation, public choice, war bargaining
- $\square$  Biology
  - evolutionary game theory, signaling and communication games
- Computer science and Logic
  - multi-agent systems, computational complexity
- □ Philosophy
  - cognitive theories (common knowledge), ethics
- $\Box$  Theology
  - Pascal's gambit

### Game Theory – Terminology

Game theory	Reality
game	conflict situation, decision situation
player	decision-maker, participant, individual, firm, political party
strategy	decision
strategy space	list of alternatives, feasible decisions
payoff	results, outcomes, consequences
rational and intelligent	maximizing utility or profit, knowing the rule of the game

### The elements of a Game

#### players

- □ number of players (2+)
- possibilities of cooperation
- existence of coalitions
- does nature/chance play a role?
- perfect/imperfect information
- strategies, strategy spaces:
  - discrete/continuous
  - simultaneous games (game 1) and sequential games (game 2)

#### payoffs

- constant-sum games vs. variable-sum games
- $\rightarrow$  different modeling techniques for different games

### Prisoner's Dilemma

- $\Box$  2 players (suspects, potential prisoners *A* and *B*)
- insufficient evidence for a conviction
- questioned separately, offered to cooperate with the police (*betray*)
- $\square$  each player 2 strategies: *betray* or *remain silent*
- $\square$  4 possible outcomes:

neither betrays	<b>1</b> year sentence for both	
A betrays	A goes free, $B$ <b>10</b> years	
B betrays	B goes free, $A$ <b>10</b> years	
both betray	both <b>5</b> years	

• what would you do if you were one them?

#### Prisoner's Dilemma

#### (cont'd)

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- □ a game in **normal** (or **strategic**) **form**
- mathematical model: a bimatrix game

	<b>A</b> \ <b>B</b>	Stay silent	Betray
Player A	Stay silent	-1,-1	-10 , 0
	Betray	<mark>0</mark> , – 10	- 5 , - 5

Player B

 $\Box$  strategy "stay silent" strictly dominated: no matter what *B* does, *A* is better off betraying him

### Prisoner's Dilemma

Real-life examples:

#### **Politics**

• arm race  $\rightarrow$  betrayal = military expansion



#### **Environment**

 $\square$  CO<sub>2</sub> emissions  $\rightarrow$  betrayal = not cutting down on emissions

□ Sport

• steroid use  $\rightarrow$  *betrayal* = *taking steroids* 

- **Economics** 
  - advertising  $\rightarrow betrayal = advertising$

Games and Decisions

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### Battle of the Sexes

- young couple dating without means of coordination (i.e., batteries gone in the mobile phones)
- preliminary discussion: two options
  - □ *football game* (meet at 6 PM by the stadium)
  - *shopping* (6 PM at the entrance of a shopping mall)
- □ model
  - another *normal form game*
  - strategies: *football* and *shopping*
  - outcome quantification: *utility*



#### Battle of the Sexes





- $\Box$  two "stable" solutions: (*F*,*F*) and (*S*,*S*)
- $\rightarrow Nash \ equilibrium \ (see \ lecture \ 2)$

#### Battle of the Sexes

#### **Modification**:

- the girl leaves earlier, decides where to go, manages to catch the boy on the phone at home, and tells him where she is
- a significant change: a *sequential* game (decisions go in turns)
- modeled as a game in **extensive form** 
  - models use decision trees



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