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Peter A. Diamond

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# Wage Determination and Efficiency in Search Equilibrium

PETER A. DIAMOND  
*Massachusetts Institute of Technology*

Using a simple search technology and the Nash bargaining solution, the paper derives the steady state equilibrium negotiated wage as a function of the equilibrium unemployment and vacancy rates. For this wage, the lifetime expected present discounted value of earnings of a new worker is compared with the social marginal product of a new worker. These are not generally equal implying inefficient incentives for labour mobility.

The concept of a competitive market is a major tool in the analysis of economists. In the simplest version of a market, resource allocation responds instantly to changes in parameters, leaving no room for frictional unemployment. One response to the unreality of this implication has been to introduce spatially distinct markets, with unemployment as workers move between markets. As modeled by Lucas and Prescott (1974), workers who are not moving are not unemployed. An alternative response is contained in the sizeable fixed price equilibrium literature, where it is assumed that prices do not change to clear markets in the short run. In contrast, the conceptual starting place of this analysis is to drop the idea of a market. Rather than markets being the mechanism by which workers and jobs are brought together, it is assumed that there is a search process which stochastically brings together unemployed workers and vacant jobs pairwise. It is taken as axiomatic that the process takes time and so involves foregone output. It is assumed that a worker and a job brought together by the search process negotiate a wage, with instantaneous negotiation. Thus, the only frictions in the model are in the search process, with wages flexible.

Actual search and negotiation processes are complicated and would be difficult to model in detail. Here we make numerous simplifying assumptions to permit explicit solution of equilibrium variables and easy analysis of their efficiency properties. The focus of the analysis is the efficiency of the incentive to enter the labour market. That is, we compare the lifetime expected present discounted value of earnings of a new worker with the social marginal product of that worker. The comparison depends critically on the bargaining solution and on the nature of the search technology. Generally, equilibrium is not efficient because of search externalities. The sources of the externalities are easy to see. The presence of an additional worker makes it easier for vacancies to find workers and harder for other workers to find jobs. The wage negotiation, however, reflects the relative bargaining powers of workers and jobs. Only in special cases will the balance of bargaining powers result in a wage which reflects the balance of search externalities as well as the value of output directly produced. The analysis identifies cases where the incentive for entry is too large or too small.

This efficiency analysis complements other efficiency analyses of search intensity, job-quitting, and job-taking (Diamond (1981), Diamond and Maskin (1979, 1981), and Mortensen (1979, 1981)). These latter papers have considered markets with equal numbers of jobs and workers. This paper focuses on the implications of unequal numbers

of jobs and vacancies. For earlier analysis of the effects of unemployment and vacancies see Holt (1970).

The paper divides into two parts. In the first five sections, we analyse the equilibrium negotiated wage as a function of the equilibrium unemployment and vacancy rates. With the assumptions used in this paper, the steady state wage depends on these rates but does not depend directly on the search technology which results in these equilibrium rates. The second part of the paper considers the search technology in relation to equilibrium and efficiency.

### 1. STEADY STATE ANALYSIS

We consider a partial equilibrium model of the labour market in steady state equilibrium. There is a fixed coefficients technology, with output of  $y$  from the combination of one worker and one job. Thus, all jobs are the same. Firms consist of a single job and workers do not organize. Unemployed workers and jobs are brought together by a stochastic process with exogenous parameters. All matches result in the start of production. In steady state equilibrium, this flow into employment is matched by an exogenous breakup of existing matches. Thus there are no economic decisions to be made (except for the trivial one to produce when able). The model focuses on wage bargains in this setting.

We begin by considering the typical worker's probability of finding a job. Denote by  $L$  the number of workers, of whom  $E$  are employed and  $U$  unemployed. We assume that there is a flow  $bE$  of workers losing their jobs, with the parameter  $b$  taken to be exogenous. With all jobs the same, there is no endogenous reason for job break up. These break ups then represent movements of jobs or workers away from each other for consumption reasons. We assume that all workers live forever. Alternatively we could have assumed a symmetric birth-death process. This would not change the expression for the equilibrium wage.

In a steady state equilibrium, we must have a flow,  $bE$ , of unemployed workers finding jobs to match the flow from breakups. If each unemployed worker has the same probability of finding a job, then each one has the flow probability  $bE/U$  of finding a job in any instant. Similarly we assume  $K$  jobs,  $F$  of which are filled and  $V$  of which are vacant. Each vacancy has the flow probability  $bE/V$  of being filled.

Since the number of filled jobs equals the number of employed workers,

$$K - V = F = E = L - U, \quad (1)$$

we have a relationship among the unemployment rate,  $u = U/L$ , the vacancy rate  $v = V/K$ , and the job-worker ratio  $k = K/L$ :

$$(1 - u) = (1 - v)k. \quad (2)$$

For the present, we use no other properties of the search process which brings together workers and jobs. For efficiency analysis below, we will need to consider the dependence of the equilibrium unemployment rate on the numbers of jobs and workers.

We assume that a filled job produces a constant flow of output,  $y$ , independent of the particular worker and job that have been matched. To begin, we assume no unemployment benefits, no disutility of labour, and no user cost of capital. The interest rate,  $r$ , is taken to be exogenous.

### 2. WAGE DETERMINATION

Next we turn to the wage bargain in this steady state setting.<sup>1</sup> That is, a theory of wage negotiations is added to the search and production technologies described above.<sup>2</sup> We assume that both firms and workers are risk neutral. This simplifies the analysis. Also, the absence of risk aversion implies that the inefficiencies found below come from search externalities given wage determination, not from the absence of insurance markets or

the imperfections of the capital market. With risk neutrality workers are interested in the expected present discounted value of wages; and firms, in the expected present discounted value of profits.

Having come together, the firm and worker have a joint surplus relative to the alternatives of waiting to find another worker and another job. The bargaining problem is to divide this surplus between them. In other words, there is a wage that makes the worker indifferent between taking this job and waiting for his next job opportunity. There is a wage that makes the firm indifferent between hiring this worker and waiting for the next available worker. The bargaining problem is to agree on a wage between these two limits. By affecting the expected times to finding the next alternatives, the unemployment and vacancy rates affect these two limits to the wage bargain. We will assume that  $u$  and  $v$  do not otherwise affect the wage bargain.

Formally, we make two assumptions on the wage bargain. First, we assume that the wage bargain is independent of the way that the worker and job have come together; that is, independent of whether the worker found the job or the job found the worker, a distinction that can only sometimes be made. This is the standard economic assumption of the irrelevance of sunk costs.<sup>3</sup> Second, we assume that the bargaining process is symmetric in the sense that the worker and job split evenly the surplus from their coming together.<sup>4</sup> Splitting the surplus evenly still leaves the wage dependent on unemployment and vacancy rates, by affecting the limits of the wage bargain. The bargained wage depends upon the anticipated wage in future employment. For equilibrium, we assume that the wage is constant over wage bargains. That is, the equilibrium wage equals the wage bargained on the assumption that future wages equal the equilibrium wage.

To derive the equilibrium wage, we need to define the surplus from coming together. For this definition, we need to describe the gain from finding (or filling) a job. We denote by  $W_U$  and  $W_E$  the expected present values of lifetime wages for unemployed and employed workers respectively. We confine analysis to steady states and assume workers live forever. With the further assumptions that the probabilities of job termination and job finding are independent of worker history these wealths are constant over workers and time. With the assumptions of risk neutrality and perfect capital markets, workers care only about these present discounted values and the surplus from finding a job is  $W_E - W_U$ . Writing the expected present discounted values of income for filled and vacant jobs as  $W_F$  and  $W_V$ , we can express the assumed symmetry in the outcome of the negotiation process as

$$W_E - W_U = W_F - W_V. \quad (3)$$

This symmetry in dividing the surplus does not imply the irrelevance of the relative availability of alternatives. If the unemployment rate is higher than the vacancy rate, firms will have to wait less long than workers for their next best alternatives. This will affect the size of the gain from making a bargain now as a function of the wage. The assumption we make is that the availability of alternatives affects the threat point of waiting for the next best alternative but does not affect the division of the surplus in excess of the sum of the two threat points. The side with the higher threat point will get the larger share of the value of output. Only the surplus is divided evenly, not the entire value of output.

Next we need to relate the expected present discounted values in (3) to the parameters of the search environment and the wage being negotiated. Treating a worker as an asset, we have the familiar condition from asset equilibrium that the rate of return times the value of being a worker equals the flow income plus the expected capital gain, or expected change in wealth.<sup>5</sup> Since we are analyzing a steady state, the only source of capital gain is the change in employment status from finding or losing a job. An unemployed worker receives no cash income and has the probability  $bE/U$  of finding a job and enjoying the

capital gain  $W_E - W_U$ . Thus we have the equation

$$rW_U = (bE/U)(W_E - W_U). \quad (4)$$

An employed worker receives a wage  $w$  and faces the exogenous probability  $b$  of suffering the capital loss  $W_E - W_U$  from losing his job

$$rW_E = w - b(W_E - W_U). \quad (5)$$

Similarly the values of filled and vacant jobs satisfy

$$rW_F = y - w - b(W_F - W_V) \quad (6)$$

$$rW_V = (bE/V)(W_F - W_V). \quad (7)$$

From the four value equations (4)–(7) and the rule describing the outcome of the negotiation process, (3), we have five equations in five unknowns, the wage and the four wealths. That is, solving these equations we find the wage which gives equal splitting of the surplus, assuming that the same wage will be reached in all future wage bargains.

### 3. EQUILIBRIUM WAGE

Before turning to the expressions for the wage in terms of the parameters of the economy and the outcome of the search process, it is interesting to combine equations (3), (4), and (7) to note that the aggregate wealth of the unemployed equals the aggregate wealth of the pool of vacancies,

$$UW_U = VW_V. \quad (8)$$

This equality rests critically on the equal shares assumption (3).

Substituting from (4)–(7) into (3) we have

$$W_E - W_U = \frac{w}{r + b + bE/U} = \frac{y - w}{r + b + bE/V} = W_F - W_V. \quad (9)$$

The extra value of current employment over unemployment depends on the wage, the rate of interest used for discounting, the probabilistic rate of termination of this employment possibility, and the probabilistic rate of finding an alternative job if current negotiations are unsuccessful ( $bE/U$ ). Similarly, the extra value from filling a job depends on profits, the rate of interest, the rate of termination of employment, and the rate of filling vacant jobs. From (9) we see that equality in the rates of finding alternatives ( $U = V$ ), results in a wage share of one-half ( $w = y - w$ ). As a special case, if there were no alternatives, the wage share would be one-half. As the interest rate rises, future alternatives become less and less important and the wage share tends to one-half. Similarly if the rate at which alternatives are found becomes small, their existence becomes less important. That is, if  $bE/U$  and  $bE/V$  both tend to zero, the wage share goes to one-half. When the break up rate becomes large (the arrival rate of alternatives held constant), it becomes likely that the current production opportunity will be over before either party receives an alternative offer. That also makes the alternatives less important for current negotiations and moves the wage share toward one-half.

Solving (9), we have the wage related to the equilibrium unemployment and vacancy rates.<sup>6</sup>

$$\frac{w}{y} = \frac{rb^{-1} + u^{-1}}{2rb^{-1} + u^{-1} + v^{-1}}. \quad (10)$$

Naturally, the wage share is lower when the unemployment rate is higher or the vacancy rate is lower, the other held constant.

## 4. LABOUR DISUTILITY AND UNEMPLOYMENT COMPENSATION

The discussion above assumed no disutility of labour and no use related costs of production. That is, the full output,  $y$ , was taken to be a return over and above the combined returns of an unemployed worker and a vacancy. We now add these elements. Assume that the disutility of labour, measured in financial terms, is  $B_U$ , or, alternatively, being unemployed adds  $B_U$  to utility. We must modify the objective function of workers to incorporate the disutility of work. Assume that workers want to maximize the expected present discounted value of wages less the monetary equivalent of the disutility of labour.

To make this modification, we rewrite the two value equations for workers to read

$$\begin{aligned} rW_E &= w - B_U - b(W_E - W_U) \\ rW_U &= (bE/U)(W_E - W_U). \end{aligned} \quad (11)$$

Equivalently we could consider workers to be maximizing the expected present discounted value of wages plus the monetary value of the utility of being idle. Taking this approach we would write the value equations as

$$\begin{aligned} rW_E &= w - b(W_E - W_U) \\ rW_U &= B_U + (bE/U)(W_E - W_U). \end{aligned} \quad (12)$$

The two approaches give the same gain from becoming employed,<sup>7</sup>  $W_E - W_U$ , and differ only in the level of measured wealth. We shall employ the latter formulation, (12), since it lends itself to the alternative interpretation that  $B_U$  is the unemployment compensation benefit.

Similarly we assume a user cost of capital,  $B_V$ , and rewrite the job value equations as

$$\begin{aligned} rW_F &= y - w - B_V - b(W_F - W_V) \\ rW_V &= (bE/V)(W_F - W_V). \end{aligned} \quad (13)$$

We continue to assume an even splitting of the surplus from filling a vacancy, (3). Thus we have five equations to determine the four wealths and the cash wage  $w$ . Solving these equations we obtain the generalized expression for the wage

$$\frac{w}{y} = \left( \frac{r + bu^{-1}}{2r + bu^{-1} + bv^{-1}} \right) \left( \frac{y - B_U - B_V}{y} \right) + \frac{B_U}{y}. \quad (14)$$

The wage equals the worker's share of the production surplus,  $y - B_U - B_V$ , plus compensation for the disutility of work or foregone unemployment benefits,  $B_U$ .

We are examining an economy where the surplus to worker and firm from production is  $y - B_U - B_V$ . As an alternative derivation of (14), let us consider an economy where  $B_U$  and  $B_V$  are zero, as above, but where the output produced,  $y^0$ , happens to equal  $y - B_U - B_V$ . In terms of private net productivity these two economies are equivalent and will have equilibria which reflect the same division of the surplus between workers and firms. The wage in this artificial economy will satisfy equation (10), with  $y^0$  replacing  $y$ . For workers to be in the same net position, the money wage in the actual economy must exceed the wage in the artificial economy,  $w^0$ , by  $B_U$ . Similarly the return to the firm exceeds its return in the artificial economy  $y^0 - w^0$ , by  $B_V$ . From these considerations, and the wage equation (10), we can move directly to the generalized wage equation

$$\frac{w - B_U}{y - B_U - B_V} = \frac{r + bu^{-1}}{2r + bu^{-1} + bv^{-1}}. \quad (15)$$

The left-hand side of (15) is the flow gain to the worker from employment as a share in the combined flow gain to firm and worker.

We can now see the role of unemployment compensation in raising wages. The availability of unemployment compensation strengthens the bargaining position of workers. The question is how much. The answer is that the decrease in the surplus from production (coming from the need to give up unemployment compensation) is divided between workers and firms in the same proportions as the surplus is divided between them. That is

$$\frac{\partial w}{\partial B_U} = 1 - \frac{w^0}{y^0} = \frac{r + bv^{-1}}{2r + bu^{-1} + bv^{-1}}. \quad (16)$$

The greater the number of vacancies relative to unemployment, the smaller the impact of unemployment compensation on wages. With many vacancies per unemployed worker, workers receive a large share of the surplus from production. With our bargaining assumption, this implies that the workers bear a large share of the fall in surplus needed to finance their own additional compensation to offset the surrender of unemployment benefits.<sup>8</sup>

### 5. WEALTH

From the wealth equations, (4) and (5), we can express the wealth of an unemployed worker in terms of the wage and the parameters of the search process (where we have again assumed  $B_U$  and  $B_V$  are zero)

$$W_U = \frac{w(1-u)b}{r(ru+b)}. \quad (17)$$

This equation can be derived alternatively by considering the probability,  $p(t)$ , of being employed at time  $t$ , as viewed from time zero. This probability satisfies the differential equation

$$\dot{p}(t) = -bp(t) + \frac{b(1-u)}{u}(1-p(t)). \quad (18)$$

The change in the probability of employment is  $-b$  if employed and  $[b(1-u)/u]$  if unemployed. Since the probabilities of employment and unemployment are  $p(t)$  and  $(1-p(t))$ , we have (18) for the change in the probability of employment. Solving (18) we have

$$p(t) = 1 - u + (p(0) - 1 + u)e^{(b/u)t}. \quad (19)$$

Since expected earnings are  $w \int e^{-rt} p(t) dt$ , we obtain (17) when  $p(0)$  is zero. For  $p(0)$  equal to one we have

$$W_E = \frac{w(ru + b(1-u))}{r(ru + b)}. \quad (20)$$

Combining the wealth equations, we have the average wealth of workers,  $uW_U + (1-u)W_E$ , equal to the discounted average wage flow  $w(1-u)/r$ . Substituting for  $w$  from (10) we can express  $W_U$  in terms of the unemployment and vacancy rates

$$W_U = \frac{(1-u)y}{r(2urb^{-1} + 1 + uv^{-1})}. \quad (21)$$

If we imbedded this market in an economy with costless labour mobility of the unemployed,  $W_U$  would be equated across markets. It is interesting therefore to compare  $W_U$  with the marginal product of labour—the present discounted value of output from the addition of a worker to a market. Thus we turn now to the details of the search technology and the effects of adding a worker to the labour pool.

## 6. MARGINAL PRODUCTIVITY

The addition of a worker to a market creates a change in the job-worker ratio and necessarily involves the movement of the economy out of a steady state. To analyze the output gain from the new path we need to specify the dynamics of the economy out of a steady state. Let us simply write the rate of matching of unemployed and vacancies as  $f(E, L, K)$ . That is,  $f$  gives the aggregate outcome of the search process. Then we have

$$\dot{E} = f(E, L, K) - bE. \quad (22)$$

With the natural assumption that  $\partial f/\partial E$  is negative, this equation is stable. Setting  $\dot{E}$  equal to zero we can determine the steady state equilibrium unemployment and vacancy rates.

We write the present discounted value of aggregate output as  $W(E, L, K)$ . That is,  $W$  measures discounted output for an economy which starts at the initial position  $(E, L, K)$  and follows the differential equation (22),  $L$  and  $K$  remaining constant.

From any initial position,

$$\begin{aligned} W(E, L, K) &= \int_0^{\infty} e^{-rt} E(t) y dt \\ \dot{E} &= f(E, L, K) - bE \\ E(0) &= E. \end{aligned} \quad (23)$$

$\partial W/\partial L$  then measures the social marginal product of an additional worker who is initially unemployed. We are interested in comparing  $\partial W/\partial L$  evaluated at a steady state equilibrium with  $W_U$ , the expected income of an unemployed worker, in that steady state. At a steady state equilibrium the marginal product of labour satisfies<sup>9</sup>

$$\frac{\partial W}{\partial L} = \left(\frac{y}{r}\right) \left(\frac{\partial f/\partial L}{r + b - \partial f/\partial E}\right). \quad (24)$$

The first term on the right-hand side is the capitalized value of output from a permanent increase in employment. If the economy adjusted instantly to the new steady state, the second term would be the change in steady state employment,  $\partial f/\partial L/(b - \partial f/\partial E)$ . However, the economy does not move instantly. Rather, it follows the differential equation (22) and the second term gives the relevant change in employment for the movement between steady states. It comes from approximating  $f$  linearly, which is equivalent to approximating the employment path exponentially. We are interested in comparing (24) with the wealth equation (21). To pursue this analysis we need to specify the search technology in more detail.

7. LINEAR TECHNOLOGY<sup>10</sup>

We begin by considering a particularly simple technology. In the next two sections we generalize this technology in two different directions. The first assumption of the linear technology is that the matching of unemployed and vacancies depends only on the numbers of unemployed and vacancies. That is, filled jobs do not affect the ability of the unemployed and the vacancies to come together. Second, both the unemployed and the vacancies are assumed to be seeking each other, with the number of matches equal to the sum of those coming from meetings initiated by workers and those initiated by jobs. The third assumption is that any searching worker makes contact with jobs at a rate independent of the number of vacant jobs (which is assumed to be strictly positive). The same condition holds for searching vacancies.

Given these assumptions, the rate of new matches is  $a_u U + a_v V$  for some positive constants  $a_u$  and  $a_v$ . That is

$$f(E, L, K) = a_v K + a_u L - (a_u + a_v)E. \quad (25)$$

With this technology, the steady state level of employment satisfies (setting (25) equal to  $bE$ )

$$E = \frac{a_v K + a_u L}{a_u + a_v + b}. \quad (26)$$

Of course this solution only holds when  $E$  is less than both  $K$  and  $L$ . Thus this solution holds when

$$b > \max [a_u(k^{-1} - 1), a_v(k - 1)], \quad (27)$$

where  $k = K/L$ . Where there is an internal solution,<sup>11</sup> adding a worker increases steady state employment by  $a_u/(a_u + a_v + b)$ , a fraction less than one. In an internal solution, the unemployment rate satisfies

$$u = 1 - \frac{E}{L} = 1 - (a_u + a_v k)/(a_u + a_v + b). \quad (28)$$

The unemployment rate is lower the greater the job-worker ratio, the longer the expected duration of a job, and the more rapid the meeting process.

Having specified a particular search technology, we can return to comparing the marginal product of a worker with expected lifetime compensation. From (24) we have

$$\frac{\partial W}{\partial L} = \left(\frac{y}{r}\right) \left(\frac{a_u}{r + a_u + a_v + b}\right). \quad (29)$$

From (21) we can write

$$W_U = \left(\frac{y}{r}\right) \left(\frac{bE/2U}{r + b + (bE/2)(U^{-1} + V^{-1})}\right). \quad (30)$$

Substituting  $a_u U + a_v V$  for  $bE$  (from the equilibrium condition) and comparing (29) and (30) we have

$$\frac{\partial W}{\partial L} \cong W_U \text{ as } a_u U \cong a_v V. \quad (31)$$

That is, workers receive more than their marginal products when they contribute less to the matching process than do vacancies.

To understand this result, let us consider the bargaining assumption analysed by Mortensen (1981).<sup>12</sup> He assumed that in a contact initiated by a worker (one of the  $a_u U$  contacts) the worker receives all of the surplus from the contact. Similarly, in each of the  $a_v V$  contacts initiated by vacancies, the firm receives all of the surplus from the contact. With the linear technology, the only effect of an individual on the search process arises from the contacts he initiates. With the initiator receiving all of the surplus from the contact, the present discounted value of wages is equal to the social marginal product. (Once we move away from the linear case, we will not generally have this equality since searchers affect others in more ways than just initiating contacts). The negotiation assumption we have employed treats the efforts that went into the contact as a sunk cost and irrelevant for the wage bargain. Thus workers receive one-half the surplus. This is more than they would receive in aggregate with Mortensen's assumption if workers initiate less than half of contacts; i.e. if  $a_u U < a_v V$ . Thus with the linear technology, expected earnings exceed the marginal product when  $a_u U < a_v V$ .

The results in (31) can be used to analyse the efficiency of labour mobility. For example, consider a country with equal aggregate number of jobs and workers and two separate regions. Assume jobs immobile and unemployed workers costlessly mobile, but able to engage in the search process in only one region. Thus  $W_U$  will be equated in the two regions. Assume that  $a_u$  equals  $a_v$  in both regions. If the only difference

between regions is that output per worker is higher in the first region,  $y_1 > y_2$ , then equilibrium will require  $k_1 < 1 < k_2$ . These inequalities imply that the marginal product of labour in region one is larger than the lifetime wage, with the reverse holding in region two. Thus we can conclude that there are more workers in the high productivity region than is efficient.

### 8. AN EXAMPLE OF NON-LINEAR SEARCH

Preserving the assumption that the rate of meetings depends only on the numbers of unemployed and vacancies, let us write the rate as a general function with positive marginal products

$$f(E, L, K) = g(U, V) = g(L - E, K - E). \quad (32)$$

That is, we are assuming that  $-\partial f/\partial E$  is equal to  $(\partial f/\partial L + \partial f/\partial K)$ .

Substituting in (24), we can write the marginal product of a worker as

$$\frac{\partial W}{\partial L} = \left(\frac{y}{r}\right) \left(\frac{\partial g/\partial U}{r + b + \partial g/\partial U + \partial g/\partial V}\right). \quad (33)$$

Substituting in (21) and using the equilibrium condition  $bE = g$ , we can write expected earnings as

$$W_U = \left(\frac{y}{r}\right) \left(\frac{g/U}{2r + 2b + g(1/U + 1/V)}\right). \quad (34)$$

If  $g$  displays constant returns to scale ( $U \partial g/\partial U + V \partial g/\partial V = g$ ) the comparison of (33) and (34) gives us the generalization of (31)

$$\frac{\partial W}{\partial L} \cong W_U \quad \text{as} \quad U \frac{\partial g}{\partial U} \cong V \frac{\partial g}{\partial V}. \quad (35)$$

Thus the generalization preserves the property that if one factor is paid more than its marginal product, the other factor is paid less. This ceases to be true with non-constant returns to scale.

With increasing returns to scale ( $U \partial g/\partial U + V \partial g/\partial V > g$ ) comparison of (33) and (34) gives the sufficient condition

$$U \frac{\partial g}{\partial U} \cong V \frac{\partial g}{\partial V} \quad \text{implies} \quad \frac{\partial W}{\partial L} > W_U. \quad (36)$$

Thus, as one would expect, an increasing returns function tends to have higher marginal products relative to earnings. Naturally, the converse holds with decreasing returns.

### 9. ANOTHER EXAMPLE OF NON-LINEAR SEARCH

It is plausible that the number of filled jobs will affect the ease with which workers can find vacancies. We can capture this idea by introducing a function of the vacancy rate,  $a_u(v)$ , to describe the ease of finding a vacancy. Plausibly  $a_u(0)$  is zero and  $a_u(v)$  increases with the vacancy rate.<sup>13</sup> Preserving the separation of search by workers and jobs, we write the outcome of the search process as<sup>14</sup>

$$\begin{aligned} f(E, L, K) &= a_u(v)U + a_v(u)V \\ &= a_u(1 - E/K)(L - E) + a_v(1 - E/L)(K - E). \end{aligned} \quad (37)$$

This form preserves the constant returns property of  $f$ . With this technology, the marginal

product of a worker satisfies

$$\frac{\partial W}{\partial L} = \left(\frac{y}{r}\right) \left(\frac{a_u + (1-u)kva'_v}{r+b+a_u+a_v+uk^{-1}a'_u+vka'_v}\right). \quad (38)$$

Expected income of an unemployed worker,  $W_U$ , does not depend on the search technology *per se*, given the equilibrium unemployment and vacancy rates. Therefore  $W_U$  in this case is the same as in the linear case with parameters  $a_u$  and  $a_v$  equal to the equilibrium levels here. For this reason, we concentrate on the comparison of  $\partial W/\partial L$  in (38) with the same expression in the linear case, (29), which equals (38) with  $a'_u$  and  $a'_v$  set equal to zero. Over time, an additional worker raises the unemployment rate and lowers the vacancy rate. The former makes it easier for jobs to find workers, the latter makes it harder for workers to find jobs. Thus, if  $a'_v$  is zero and  $a'_u$  positive, the latter effect dominates and the marginal product of a worker is lower in the non-linear case. If  $a'_u$  is zero and  $a'_v$  positive, the former effect dominates and the marginal product of a worker is larger in the non-linear case. (From (28),  $1-u > a_u/(r+b+a_u+a_v)$ , which gives the result from inspection of (38).) In the symmetric case ( $k=1$ ,  $a_u(z) = a_v(z)$ ) the latter effect dominates and the marginal product of a worker is larger in the non-linear case. (From (28)  $(1-u/2 > a_u/(r+b+a_u+a_v))$ , which gives the result from inspection of (38).) In the symmetric case both firms and workers are paid less than their marginal products.

The absence of an instantaneous resource allocation mechanism (like the Walrasian auctioneer) creates a strictly positive surplus for any deal compared with the next best alternative. (In this sense there is no consumer surplus in Walrasian competitive equilibrium.) The division of this surplus between trading partners can occur in a number of different institutional settings. Here we have explored one particular setting—symmetric bargaining. With this method of determining trading prices, these prices are not generally an efficient incentive for behaviour affecting search, such as entry into a market. In addition we found that unemployment compensation affects wages even when it does not affect the rate at which workers accept jobs.

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#### NOTES

1. We do not consider wages set by one side on a take-it-or-leave-it basis.
2. Mortensen (1979) has taken an alternative approach of relating market outcome to bargaining outcomes rather than specifying a single equilibrium concept.
3. Mortensen (1981) explores the alternative approach of having the wage bargain depend on the source of the match.
4. Alternative splitting rules are considered below.
5. Of course, there is no market for workers as assets. This equation can be derived alternatively by differentiating the expression for expected earnings with respect to time,  $d/dt W_U(t) = rW_U - (bE/U) \times (W_E - W_U) = 0$ .
6. If workers receive  $s/(1+s)$  of the surplus,  $W_E - W_U = s(W_F - W_V)$ , then (10) becomes  $w/y = s(rb^{-1} + u^{-1}) / ((1+s)rb^{-1} + su^{-1} + v^{-1})$ .
7.  $W_E - W_U = (w - B_U)(r+b + bE/U)^{-1}$ .
8. In a full equilibrium analysis, unemployment compensation would also affect the number and productivity of jobs.
9. For a derivation of (24), see Diamond (1980).
10. The linear technology has been analysed by Diamond and Maskin (1979, 1981) and by Mortensen (1979, 1981).
11. We do not analyse the corner solutions since the linear technology is not plausible with an unemployment or vacancy rate close to zero. Particularly the third assumption used in deriving the linear technology is implausible.
12. This interpretation was given to me by Mortensen.

13. For example  $a_u$  might have the form  $a(1+e)v/(1+ev)$ . This form would arise if job search were akin to an urn problem (sampling with replacement) with 1 ball per filled job and  $1+e$  balls per vacancy.

14. Note that we no longer satisfy the condition  $-\partial f/\partial E = \partial f/\partial L + \partial f/\partial K$ . That is, the search outcome can no longer be written solely in terms of the numbers of unemployed and vacancies.

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